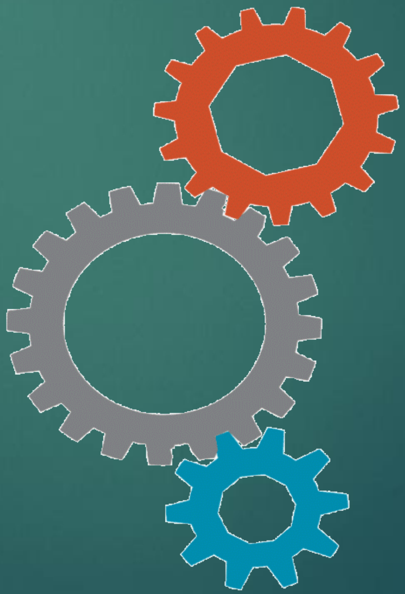
Gears and Gear Trains

Introduction

Gears are used to transmit motion from one shaft to another or between a shaft and a slide. This is accomplished by successfully engaging teeth.

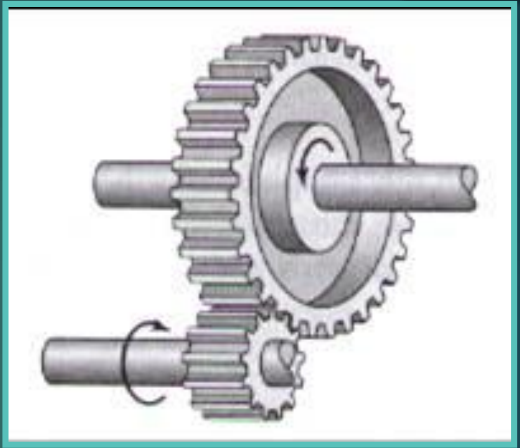


Types of Gears

1. Parallel Shafts

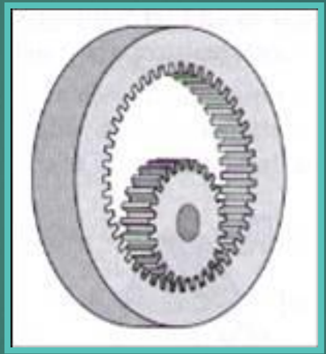
Spur gears – tooth profile is parallel to the axis of rotation, transmits motion between parallel shafts.

Gear (large gear)

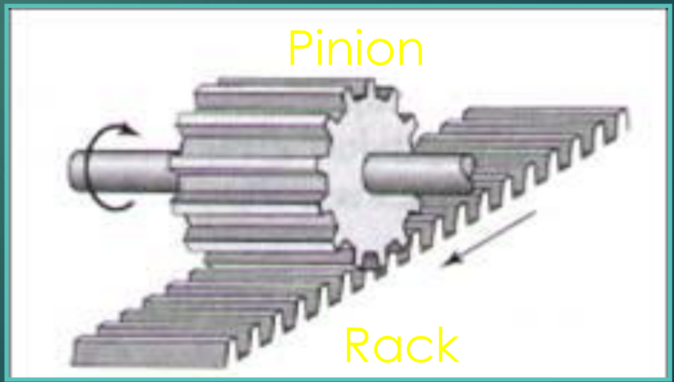


Pinion (small gear)

Internal gears

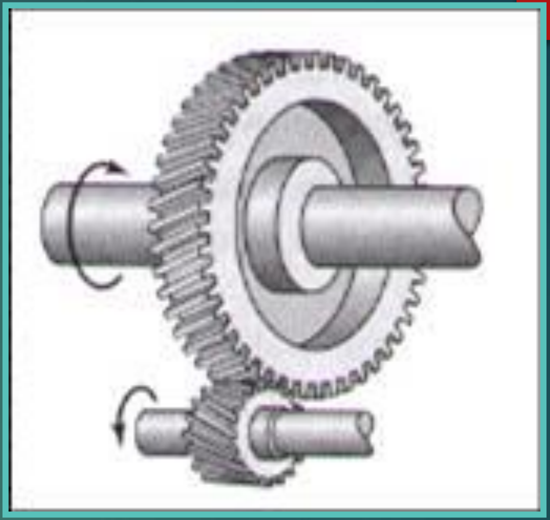


Spur Rack and Pinion sets – a special case of spur gears with the gear having an infinitely large diameter, the teeth are laid flat.

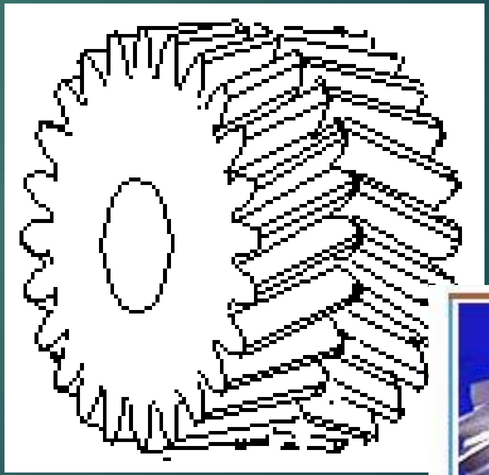


1. Parallel Shafts

Helical gears– teeth are inclined to the axis of rotation, the angle provides more gradual engagement of the teeth during meshing, transmits motion between parallel shafts.



- **Herringbone gears-** To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces



- Herringbone gears are mostly used on heavy machinery.

2. Intersecting shafts

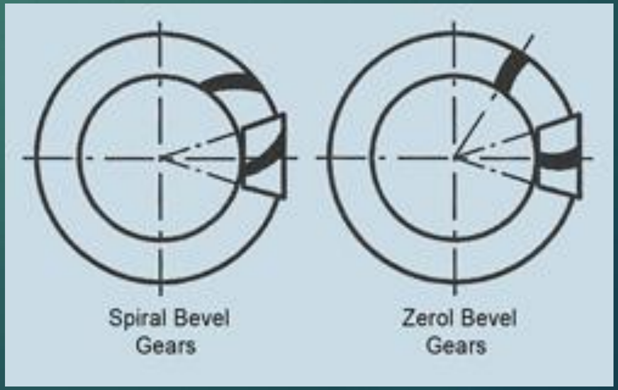
Bevel gears – teeth are formed on a conical surface, used to transfer motion between non-parallel and intersecting shafts. **Straight bevel gears** make line of contact similar to spur gears

Straight bevel gear

Spiral bevel gears- smoother in action and quieter than straight bevel gears.

Spiral bevel gear

Zero Bevel Gear- In this bevel gear spiral angle is zero at the middle of the face width



3. Skew shafts (non- parallel and non-intersecting)

In case of Skew shafts (Non- parallel- non-intersecting) a uniform rotary motion is not possible as in case of parallel and intersecting shafts which has pure rolling contact

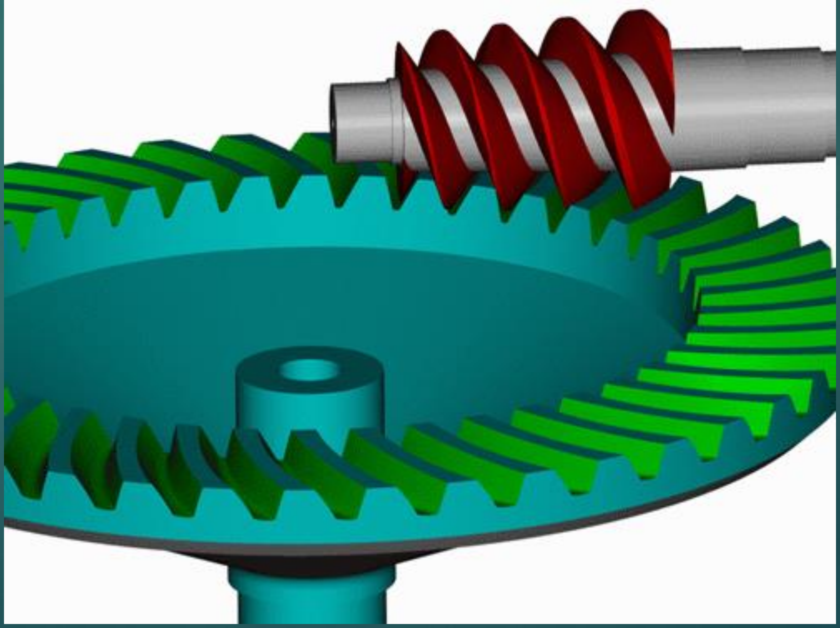
Worm gear sets – consists of a helical gear and a power screw (worm), used to transfer motion between non-parallel and non-intersecting shafts.

Crossed Helical gears- Applicable to light load conditions. Used to drive feed mechanisms in machine tools, camshafts, and small IC engines

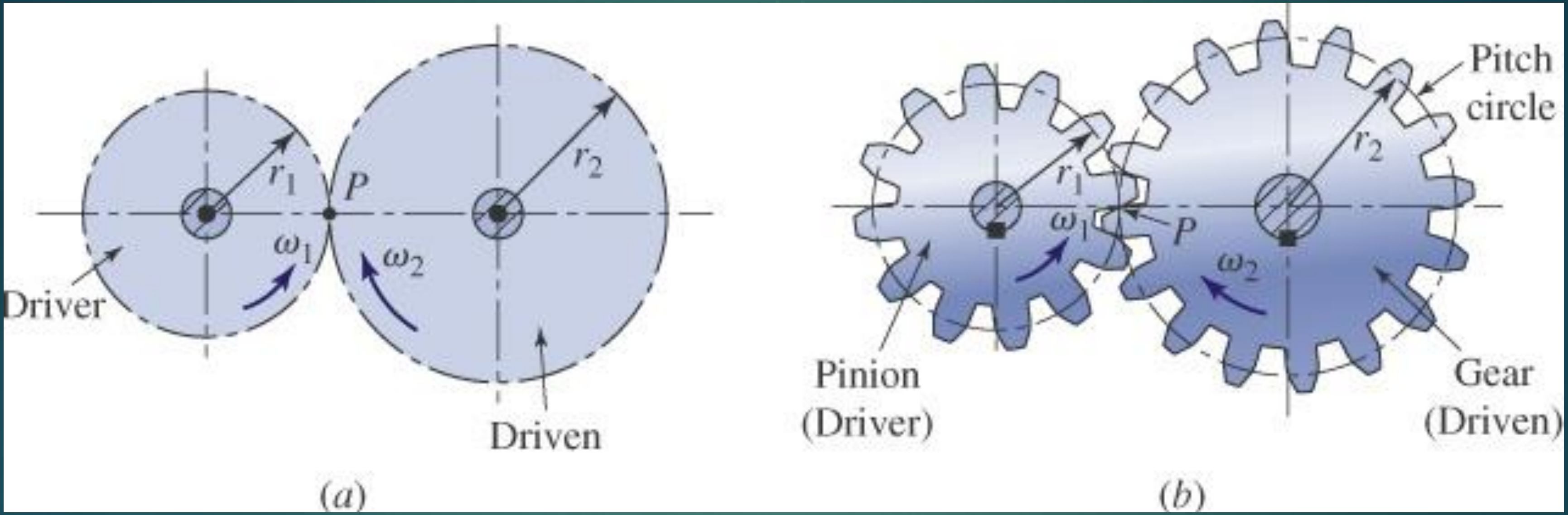
3. Skew shafts (non- parallel and non- intersecting)

Hypoid gears

Bevel gears have straight teeth very similar to spur gears. Modified bevel gears having helical teeth are known as hypoid gears. The shafts of these gears, although at right angles, are not in the same plane and, therefore, do not intersect. Hypoid gears are used in automobile drives.

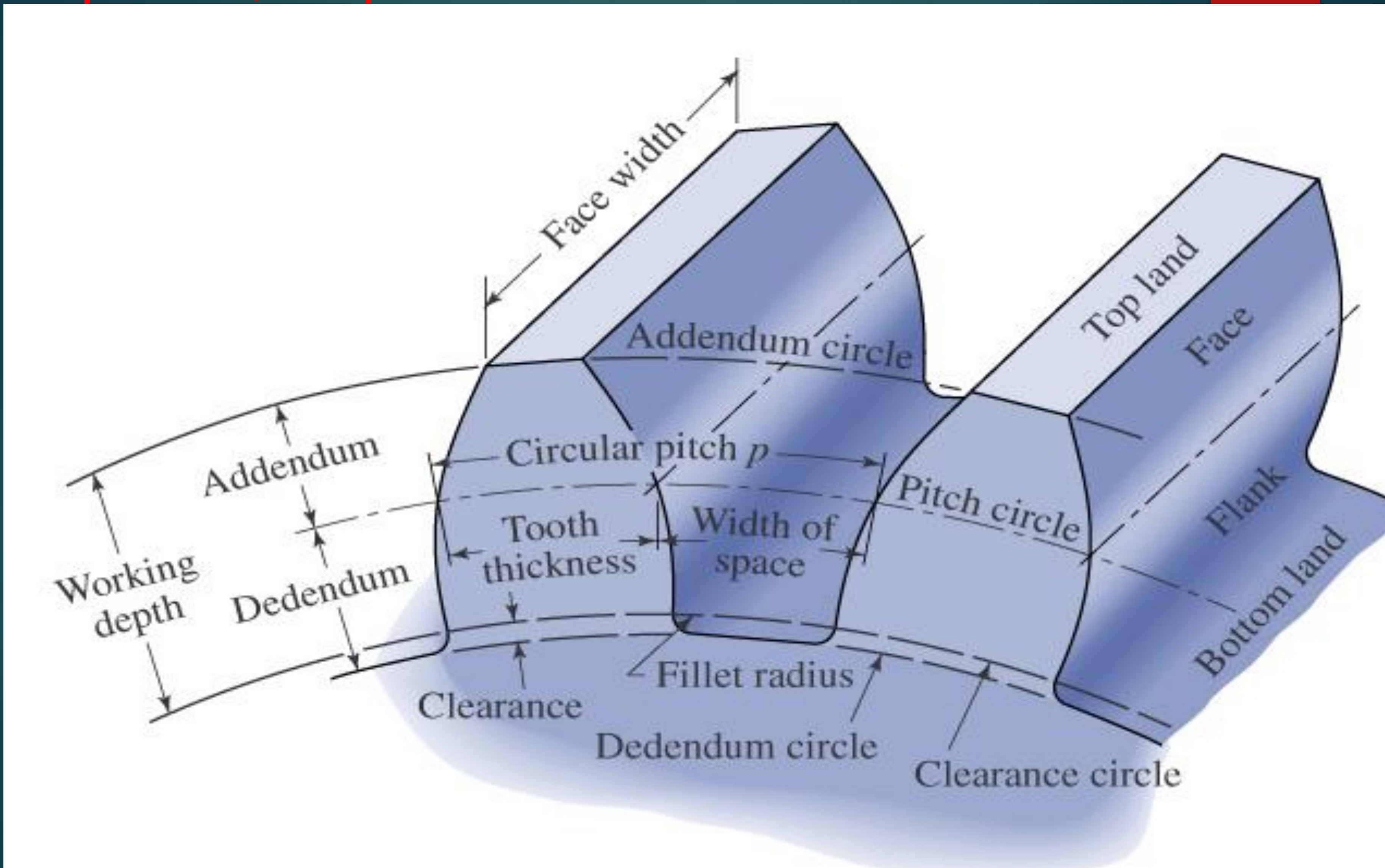


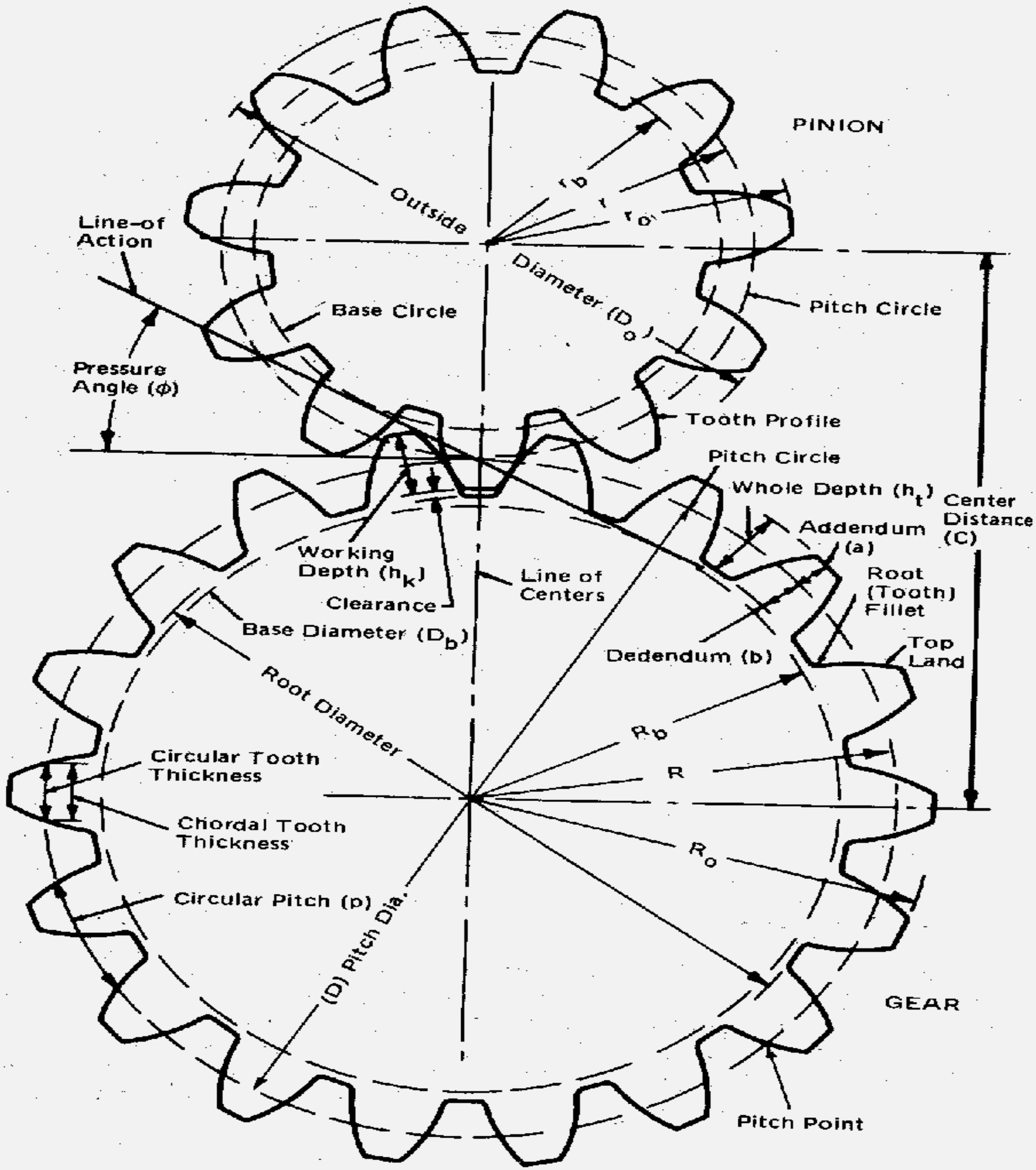
Nomenclature



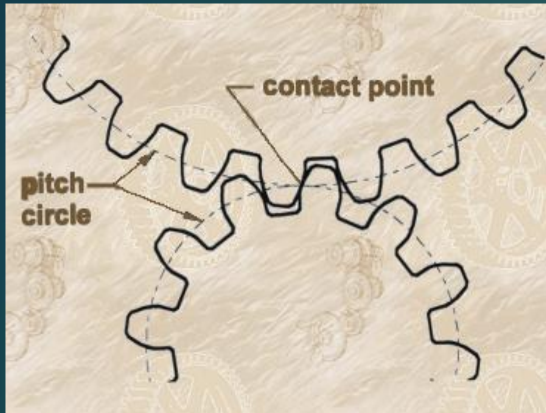
Smaller Gear is Pinion and Larger one is the gear

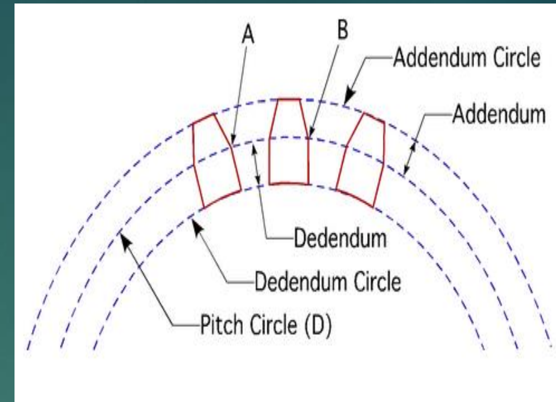
In most application the pinion is the driver, This reduces speed but it increases torque.


Face width
Top land
Face
Addendum circle
Addendum
Circular pitch p
Pitch circle
Flank
Working depth
Tooth thickness
Width of space
Bottom land
Dedendum
Fillet radius
Clearance
Dedendum circle
Clearance circle



PINION
Outside
Diameter (D_o)
r_b
r
r_o
Line-of Action
Base Circle
Pitch Circle
Pressure Angle (ϕ)
Tooth Profile
Pitch Circle
Whole Depth (h_t)
Center Distance (C)
Addendum (a)
Working Depth (h_k)
Clearance
Line of Centers
Root (Tooth) Fillet
Base Diameter (D_b)
Dedendum (b)
Top Land
Root Diameter
R_b
R
Circular Tooth Thickness
Chordal Tooth Thickness
R_o
Circular Pitch (p)
(D) Pitch Dia.
GEAR
Pitch Point





Pitch circle, theoretical circle upon which all calculation is based

P_c, Circular pitch is the distance from one teeth to the next, along the pitch circle.

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

Note : If *D*1 and *D*2 are the diameters of the two meshing gears having the teeth *T*1 and *T*2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

m, module=d/T , pitch circle diameter/number of teeth

Therefore Circular pitch, $P_c = \pi m$

P_d, Diametral Pitch , Number of teeth per unit length,

P_d =T/D= t/d=1/m

$P_c P_d = \pi$

Backlash

It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

Velocity Ratio (VR)

The velocity ratio is defined as the ratio of the angular velocity of the driven gear to the angular velocity of the driving gear

$VR = \omega_2 / \omega_1$ (D= diameter of driven gear; d= diameter of driving gear)

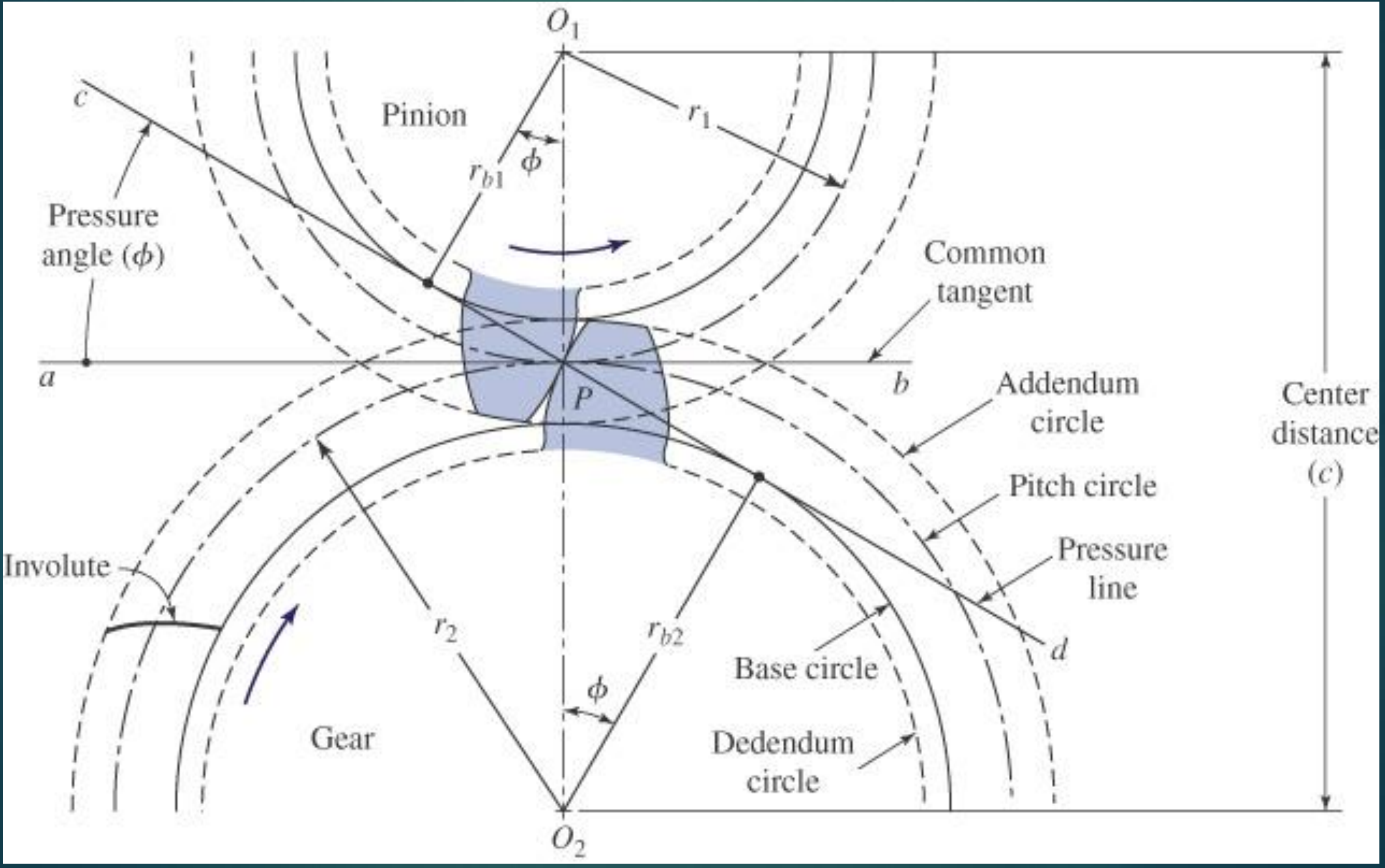
$= N_2 / N_1$ $(\omega = 2\pi N)$

$= d / D$ $(V = \pi d N_1 = \pi D N_2)$

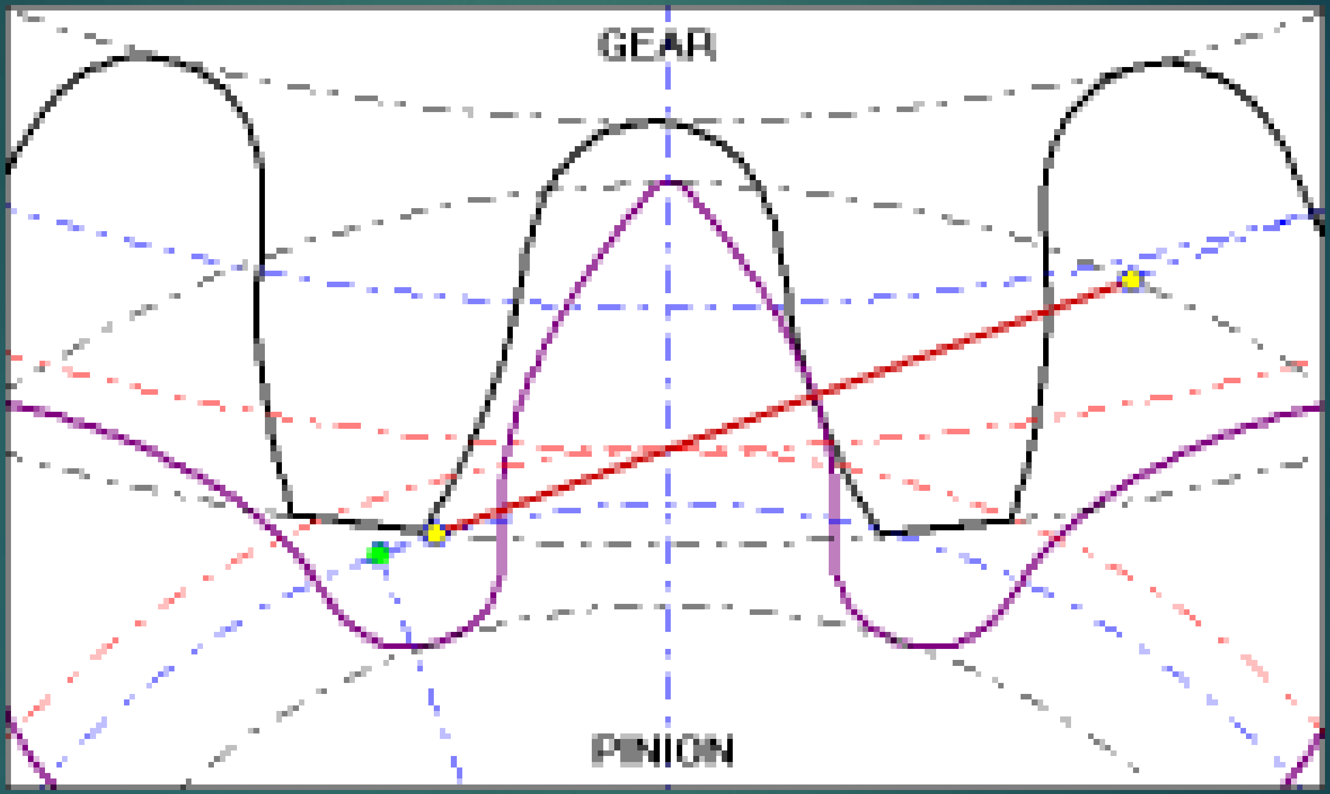
$= t / T$ $(p = \pi d / T_1 = \pi D / T_2)$

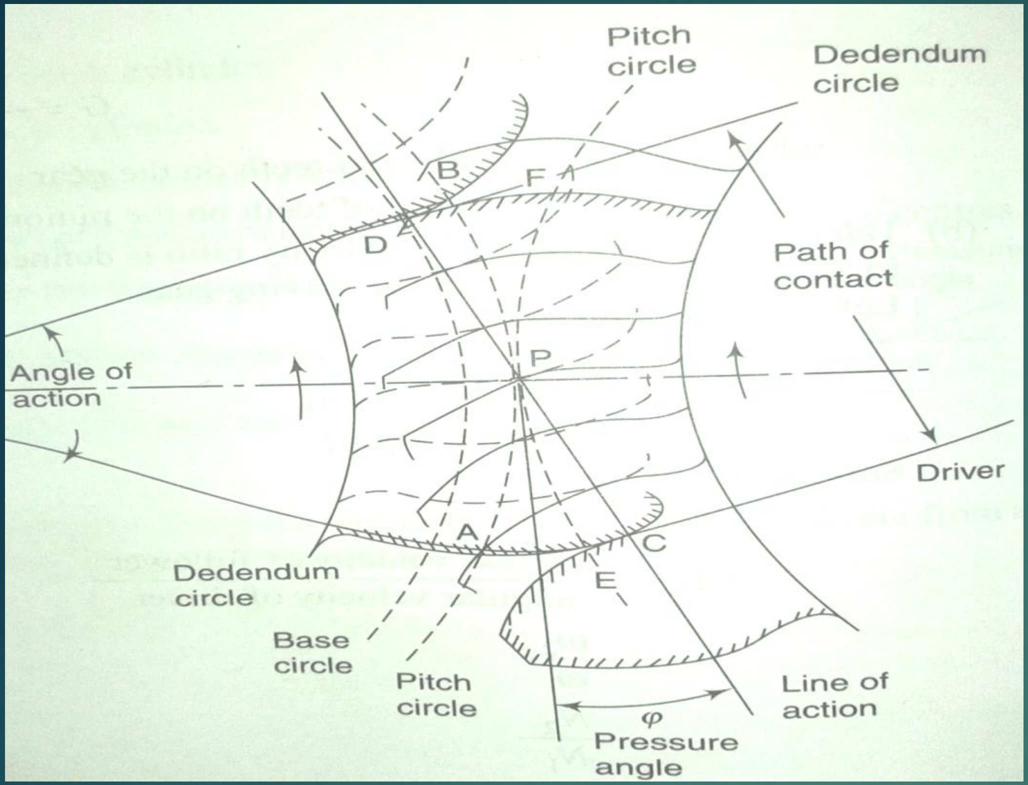
Pressure line and pressure angle

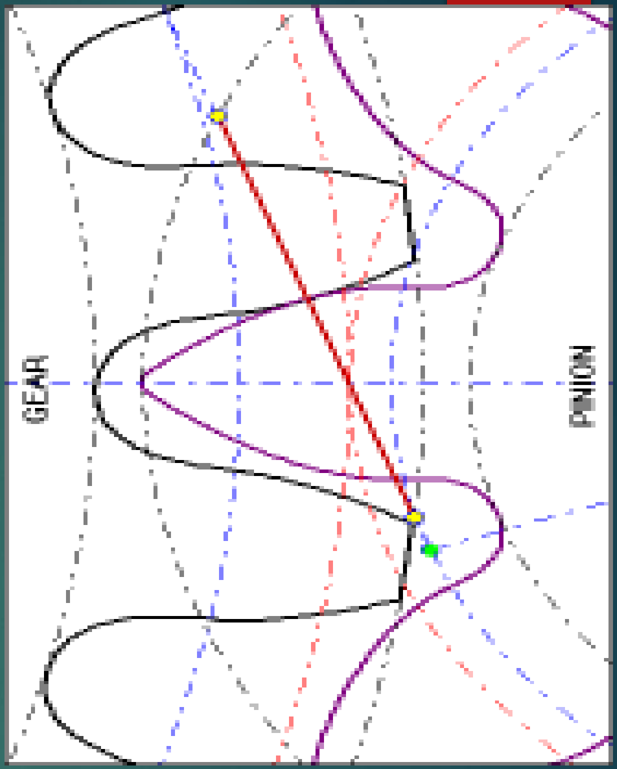
- The standard pressure angles are 14 1/2 ° and 20°



Pressure line







CP- path of approach
PD- Path of recess
Arc of contact- arc APB or EPF
Arc of approach- AP or EP
Arc of recess- PB or PF
Angle of action (δ)= Angle of approach(α) + Angle of recess (β)

$$\text{Contact ratio} = \frac{\text{Angle of action } (\delta)}{\text{Pitch angle } (\gamma)} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

[Pitch angle (γ) - Angle subtended by circular pitch at the center of gear]

Contact ratio

Number of pairs of teeth in contact

$$\text{Contact ratio} = \frac{\text{Angle of action } (\delta)}{\text{Pitch angle } (\gamma)} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

Contact ratio should be always greater than unity to ensure continuous transmission of motion, for at least one pair of teeth should be in meshed condition for the mating gears.

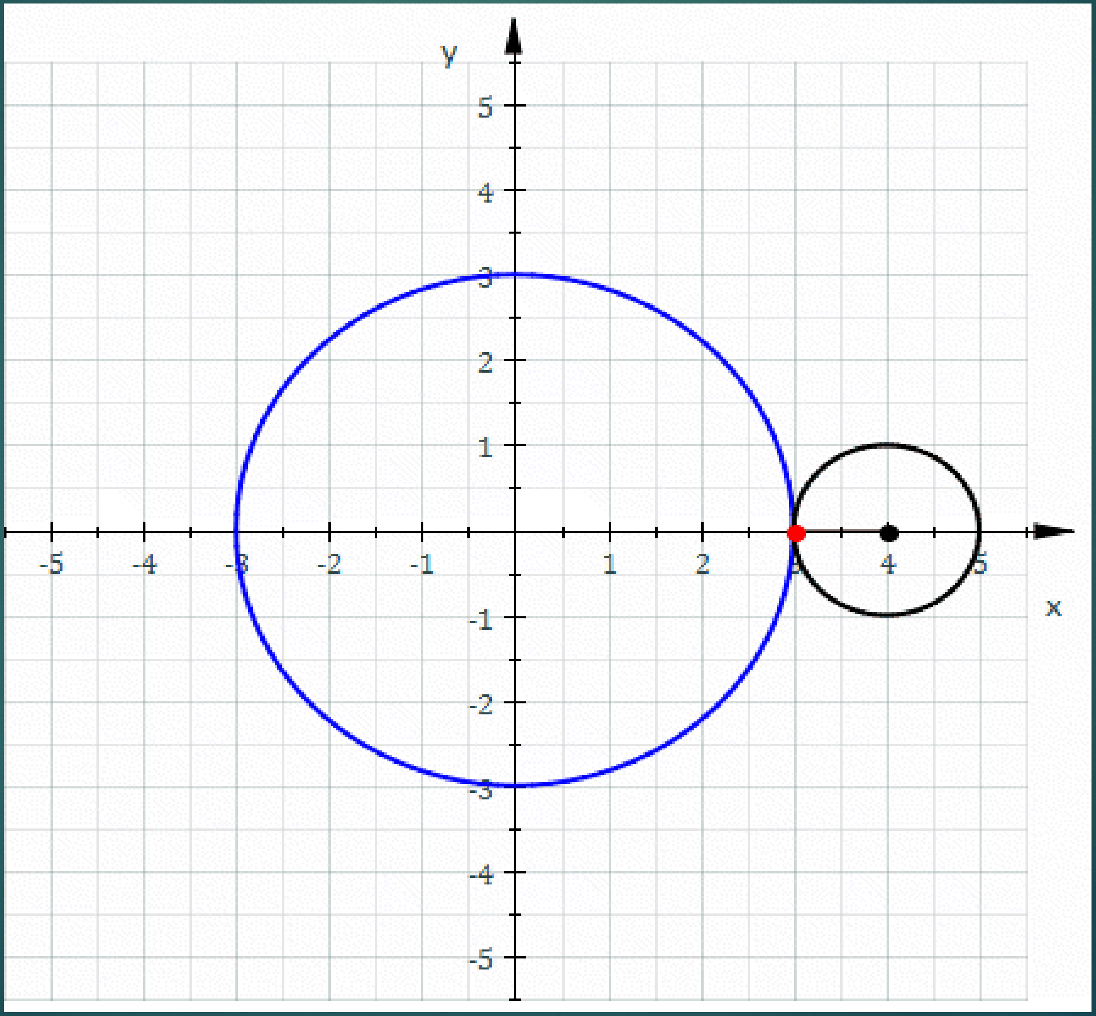
If contact ratio is 1.6, it means that one pair of teeth is always in contact whereas two pairs of teeth are in contact for 60% of the time.

FORMS OF TEETH

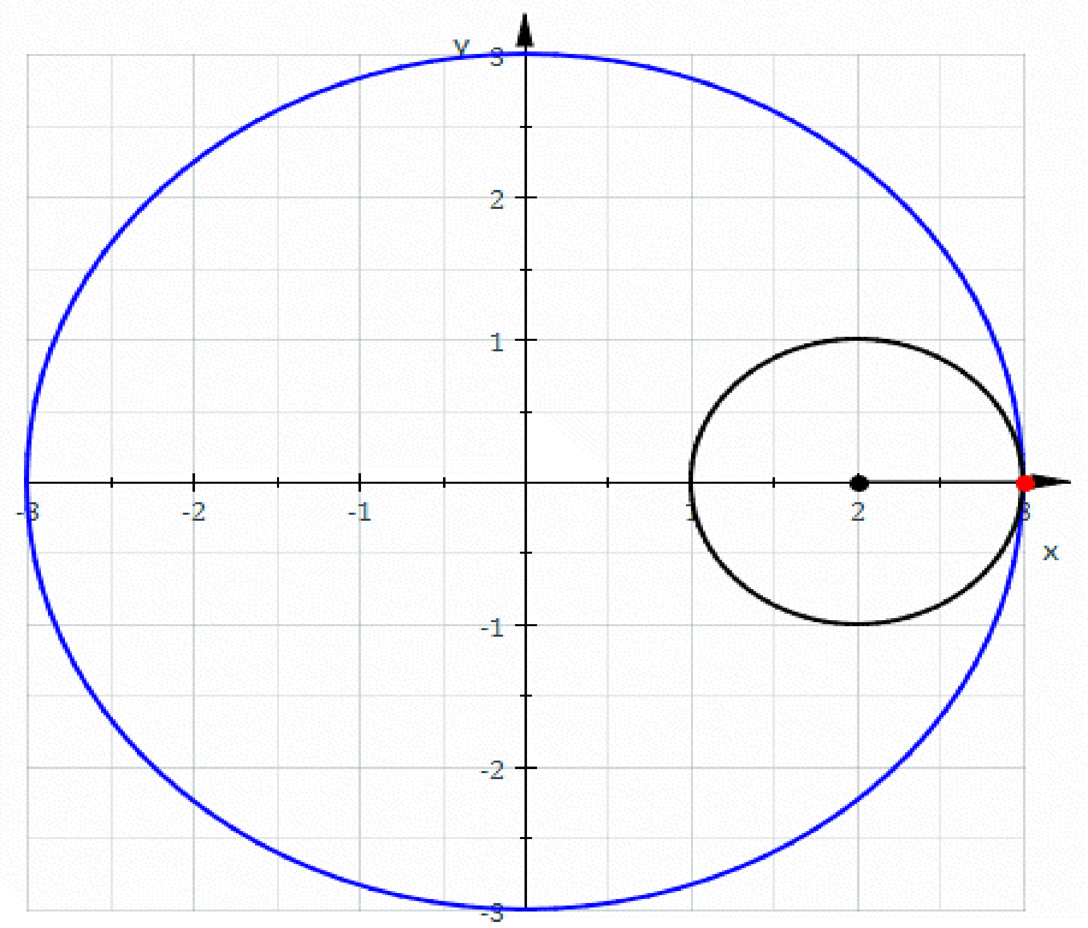
1.Cycloidal profile
2.Involute profile

Cycloid

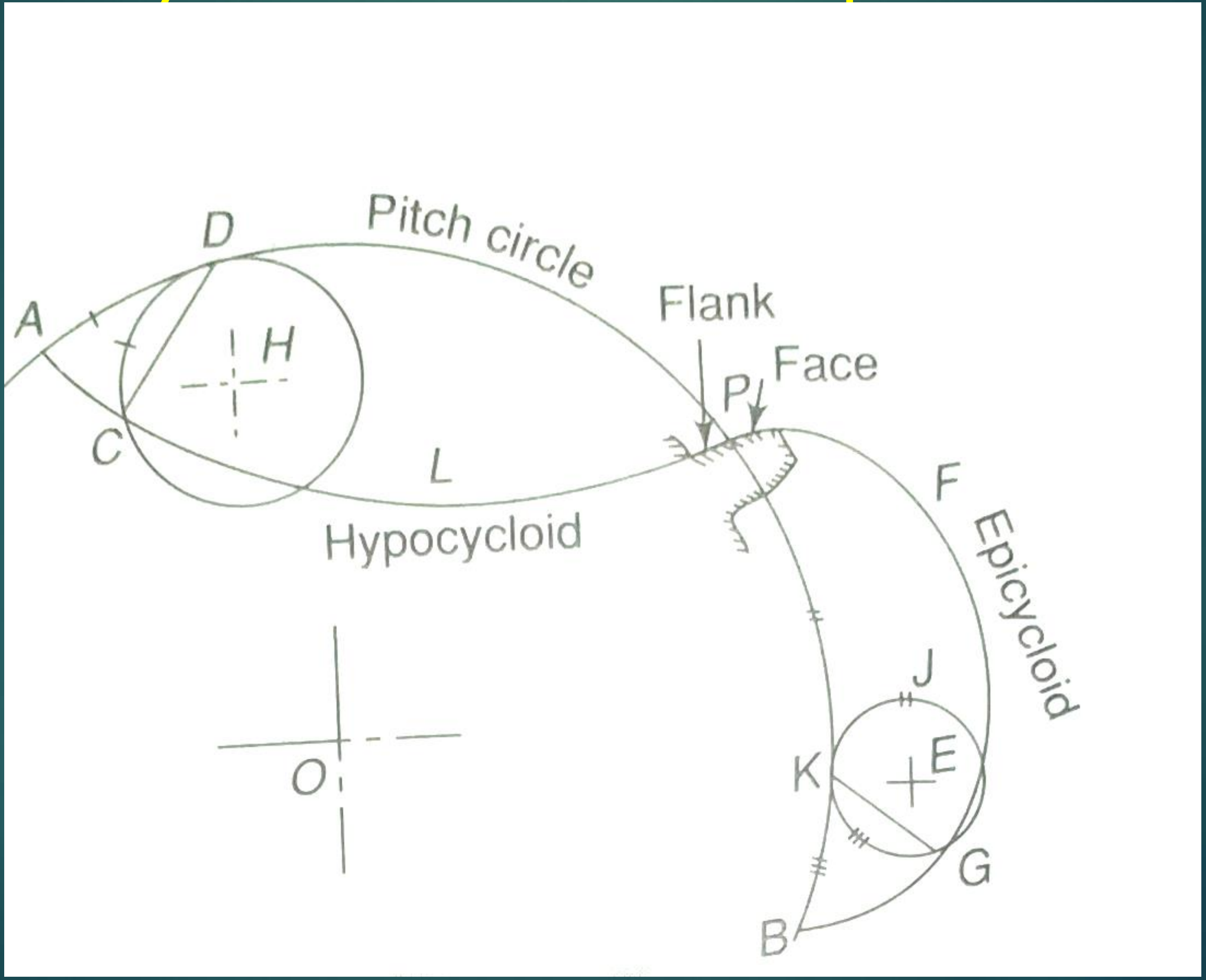
Epicycloid



Hypocycloid



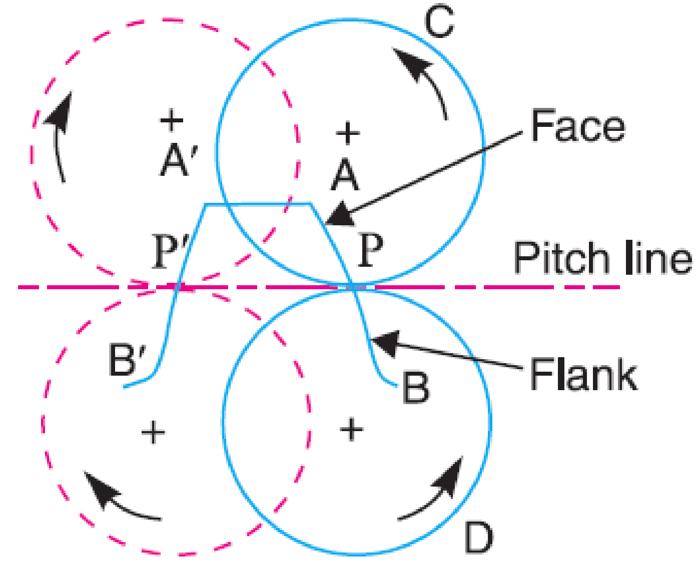
Epicycloidal and hypocycloidal teeth profile



Construction of cycloidal teeth for Rack

- In Fig. *(a), the fixed line or pitch line of a rack is shown. When the circle C rolls without* slipping above the pitch line in the direction as indicated in Fig. *(a), then the point P on the circle* traces epi-cycloid *PA. This represents the face of the cycloidal tooth profile.*
- *When the circle D rolls* without slipping below the pitch line, then the point *P on the circle D traces hypo-cycloid PB, which* represents the flank of the cycloidal tooth. The profile *BPA is one side of the cycloidal rack tooth.*

Similarly, the two curves *P' A' and P'B' forming the opposite side of the tooth profile are traced by* the point *P' when the circles C and D roll in the opposite directions.*

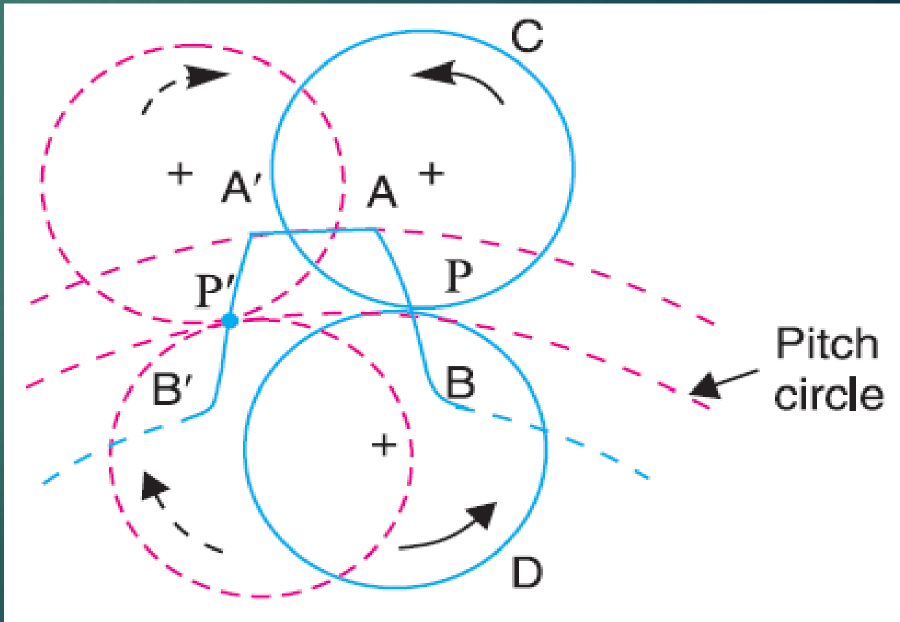


(a)

Construction of cycloidal teeth for gear

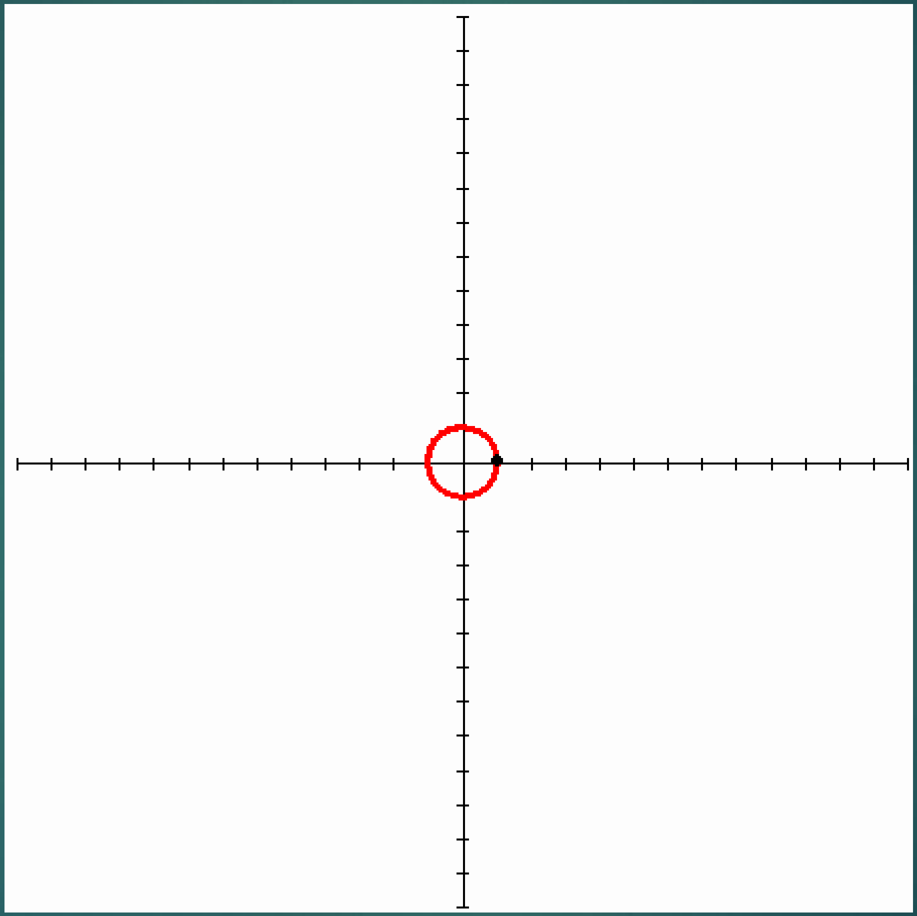
- In the similar way, the cycloidal teeth of a gear may be constructed as shown in Fig. *(b)*.
- The circle C *is rolled without slipping on the outside of the pitch circle and the point P on the circle C traces epi-cycloid PA, which represents the face of the cycloidal tooth.*
- *The circle D is rolled on the* inside of pitch circle and the point *P on the circle D traces hypo-cycloid PB, which represents the* flank of the tooth profile.

The profile *BPA is one side of the cycloidal tooth. The opposite side of the* tooth is traced as explained above.



(b)

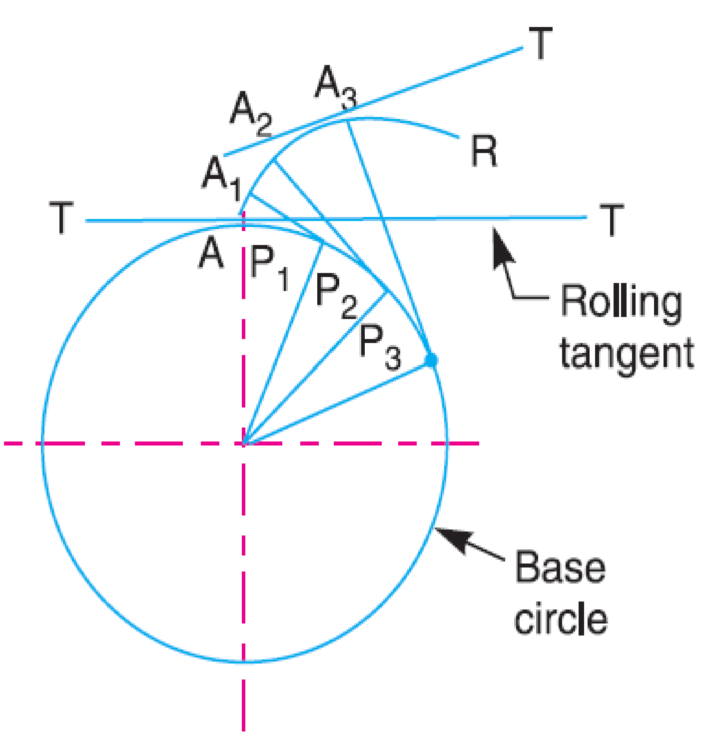
Involute profile



Involute Teeth

- An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig.
- In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :

Let A be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, P_1P_2, P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve AR. A little consideration will show that at any instant



- *A3, the tangent A3T to the involute is perpendicular to P3A3* and *P3A3* is the normal to the involute.
- In other words, ***normal at any point of an involute is a tangent to the circle.***

Comparison Between Involute and Cycloidal Gears

- In actual practice, the involute gears are more commonly used as compared to cycloidal gears, due to the following advantages :

Advantages of involute gears

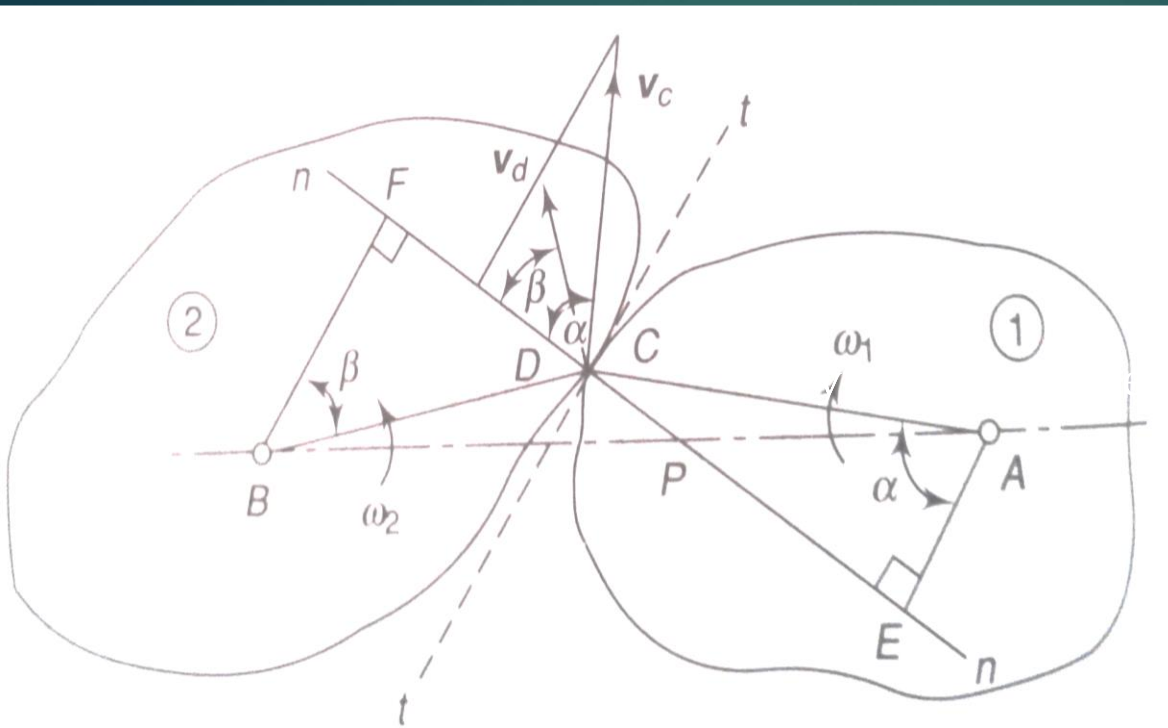
- The most important advantage of the involute gears is that **the centre distance for a pair of involute gears can be varied within limits** without changing the velocity ratio. This is not true for cycloidal gears which requires exact centre distance to be maintained.
- **In involute gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant**. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts decreasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.
- The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (*i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively*. Thus the **involute teeth are easy to manufacture than cycloidal teeth**. In involute system, the basic rack has straight teeth and the same can be cut with simple tools.
- Note : **The only disadvantage of the involute teeth is that the interference occurs with pinions having smaller number of teeth**. This may be avoided by altering the heights of addendum and dedendum of the mating teeth or the angle of obliquity of the teeth.

Advantages of cycloidal gears

Following are the advantages of cycloidal gears :

- Since the cycloidal teeth have wider flanks, therefore the **cycloidal gears are stronger than the involute gears**, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.
- In cycloidal gears, the contact takes place between a convex flank and concave surface, whereas in involute gears, the convex surfaces are in contact. This condition results in **less wear in cycloidal gears as compared to involute gears**. However the difference in wear is negligible.
- **In cycloidal gears, the interference does not occur at all**. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

Law of Gearing



$$v_c \cos\alpha - v_d \cos\beta = 0$$

$$\omega_1 AC \cos\alpha - \omega_2 BD \cos\beta = 0$$

$$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

$$\omega_1 AE - \omega_2 BF = 0$$

Component of v_c along $n-n = v_c \cos\alpha$

Component of v_d along $n-n = v_d \cos\beta$

Relative motion along $n-n = v_c \cos\alpha - v_d \cos\beta$

$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP} = \frac{FP}{EP}$$

Velocity of sliding

If the curved surfaces of the two teeth of the gears 1 and 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent $t-t$ at C or D

Component of v_c along $t-t = v_c \sin\alpha$

Component of v_d along $t-t = v_d \sin\alpha$

Velocity of sliding $= v_c \sin\alpha - v_d \sin\beta$

$$= \omega_1 AC \frac{EC}{AC} - \omega_2 BD \frac{FD}{BD}$$

$$= \omega_1 EC - \omega_2 FD$$

$$= \omega_1 (EP + PC) - \omega_2 (FP - PD)$$

$$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PC$$

(C and D are the coinciding points)

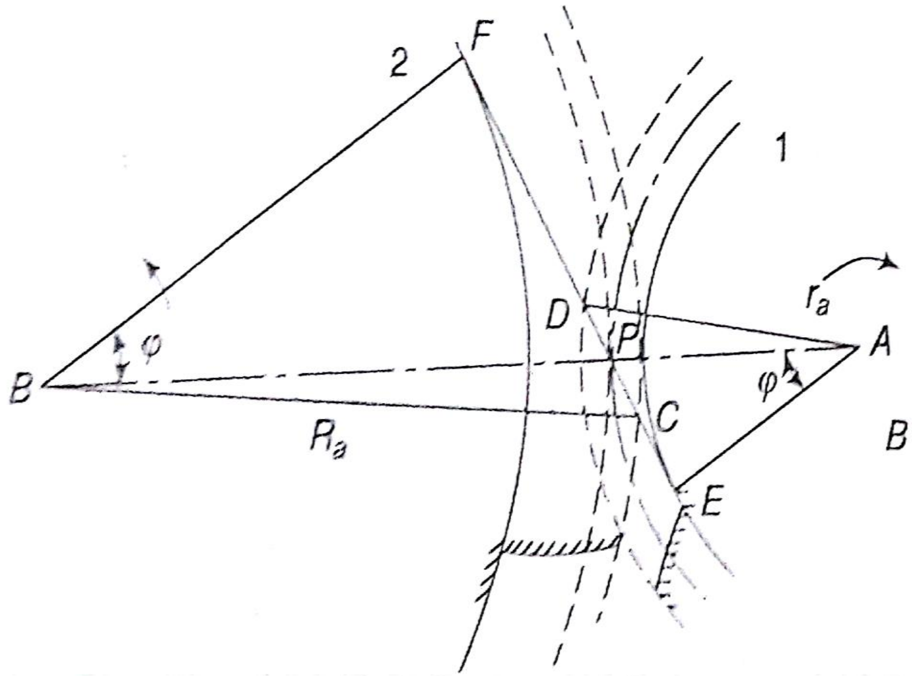
$$= (\omega_1 + \omega_2)\, PC + \omega_1 EP - \omega_2 FP$$

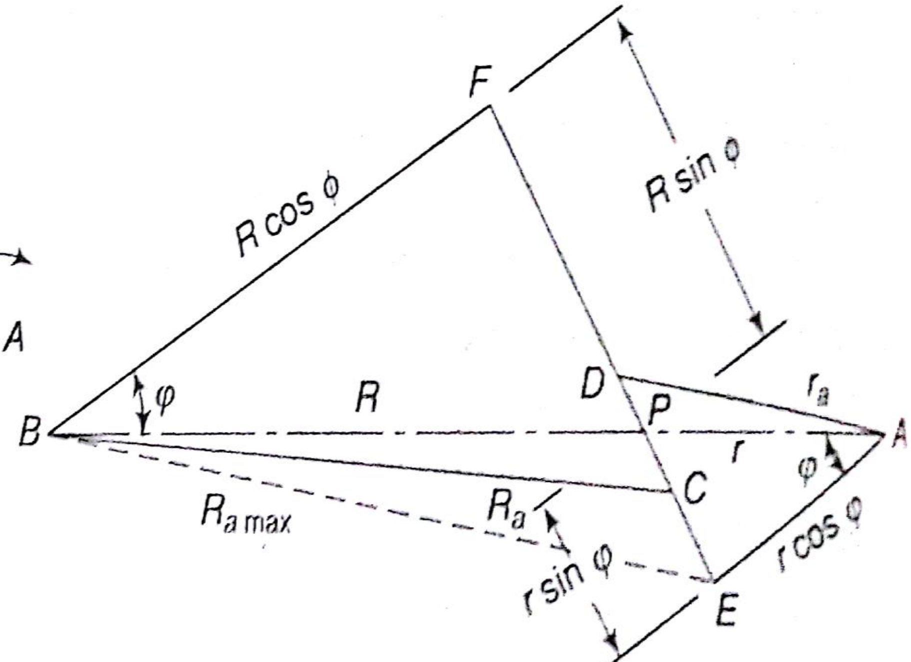
$$= (\omega_1 + \omega_2)\, PC$$

$\omega_1 EP = \omega_2 FP$, Law of gearing

= sum of angular velocities × distance between the pitch point and the point of contact

Path of contact





Let r = pitch circle radius of pinion

R = pitch circle radius of wheel

r_a = addendum circle radius of pinion

R_a = addendum circle radius of wheel.

Path of contact = path of approach + path of recess

$$CD = CP + PD$$

$$= (CF - PF) + (DE - PE)$$

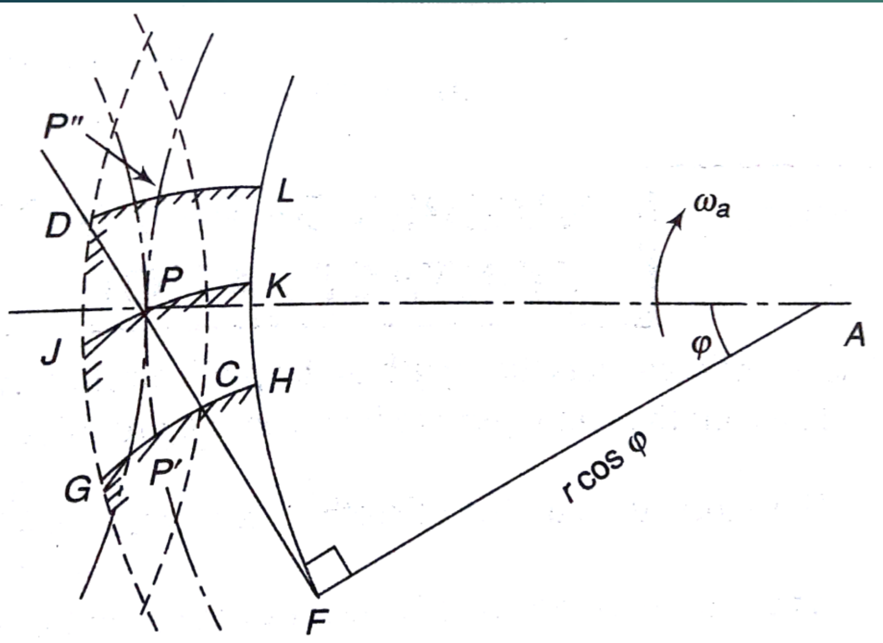
$$= \left[\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\right]$$

$$= \sqrt{R_a^2 - R^2\cos^2\varphi} + \sqrt{r_a^2 - r^2\cos^2\varphi} - (R + r)\sin\varphi$$

Arc of contact

Arc of approach PP' + Arc of recess PP"

Arc of approach PP'



Let the time to traverse the arc of approach is t_a.

Then Arc of approach = $P'P$ = Tangential velocity of P' × Time of approach

$$= \omega_a r \times t_a$$

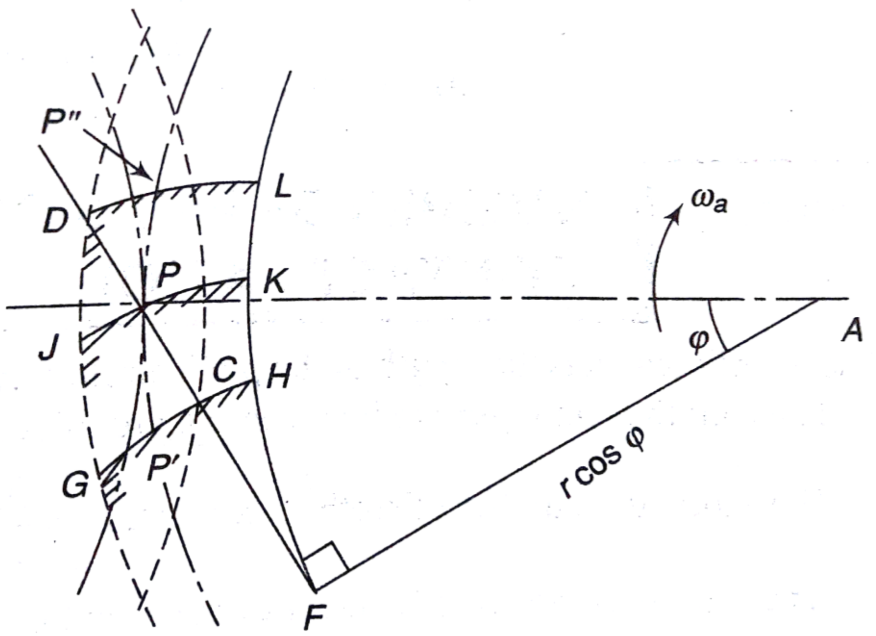
$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi} t_a$$

$$= (\text{Tang. vel. of } H) t_a \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } HK}{\cos\varphi} = \frac{\text{Arc } FK - \text{Arc } FH}{\cos\varphi}$$

$$= \frac{FP - FC}{\cos\varphi} = \frac{CP}{\cos\varphi}$$

Arc of recess PP''



$$PP'' = \text{Tang. vel. of } P \times \text{Time of recess}$$

$$= \omega_a r \times t_r$$

$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi} t_r$$

$$= (\text{Tang. vel. of } K)\, t_r \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } KL}{\cos\varphi} = \frac{\text{Arc } FL - \text{Arc } FK}{\cos\varphi}$$

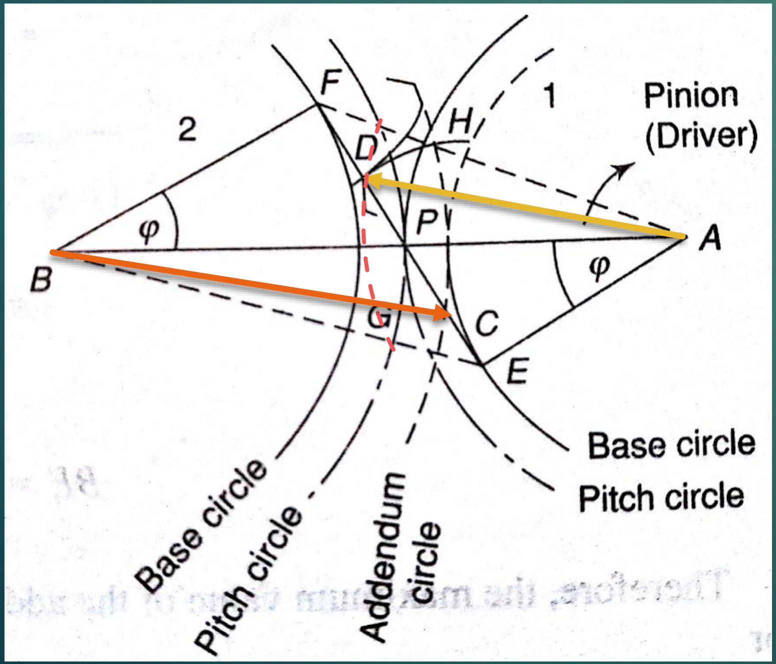
$$PP'' = \frac{FD - FP}{\cos\varphi} = \frac{PD}{\cos\varphi}$$

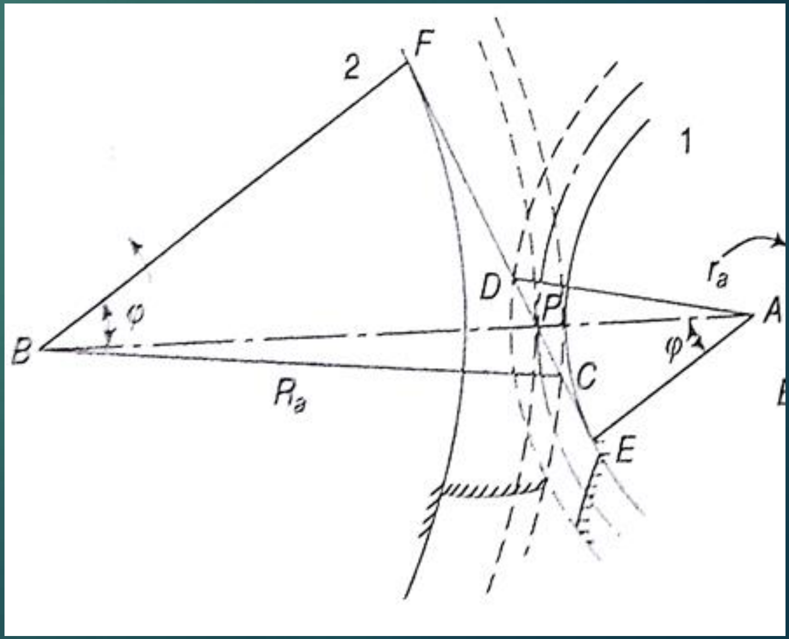
$$\text{Arc of contact} = \frac{CP}{\cos\varphi} + \frac{PD}{\cos\varphi} = \frac{CP + PD}{\cos\varphi} = \frac{CD}{\cos\varphi}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$

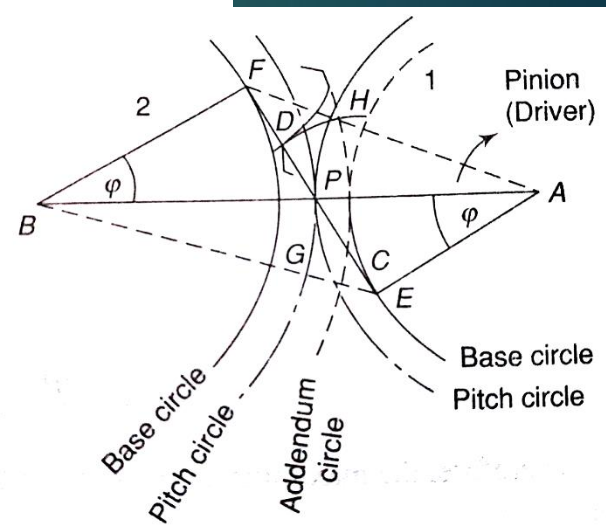
Interference in involute gears

- Mating of two non- conjugate (non- involute)teeth is known as interference
- Two teeth does not slide properly and thus rough action and binding occurs
- Contacting teeth have different velocities which can lock the two gears
- The points E and F are called interference points





$$(BE)^2 = (BF)^2 + (FE)^2$$
$$= (BF)^2 + (FP + PE)^2$$
$$= (R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2$$
$$= R^2\cos^2\varphi + R^2\sin^2\varphi + r^2\sin^2\varphi + 2\,r\,R\sin^2\varphi$$
$$= R^2(\cos^2\varphi + \sin^2\varphi) + \sin^2\varphi\,(r^2 + 2rR)$$
$$= R^2 + (r^2 + 2rR)\sin^2\varphi$$
$$= R^2\left[1 + \frac{1}{R^2}(r^2 + 2rR)\sin^2\varphi\right]$$
$$= R^2\left[1 + \left(\frac{r^2}{R^2} + \frac{2r}{R}\right)\sin^2\varphi\right]$$
$$BE = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi}$$



Maximum possible Addendum value for teeth to avoid interference

$$a_{w\,max} = R\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\varphi} - R = R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\varphi} - 1\right]$$

t = number of teeth on the pinion
T = number of teeth on the wheel

$$R = \frac{mT}{2},\ r = \frac{mt}{2} \quad \text{and} \quad G = \frac{T}{t} = \text{Gear ratio}$$

$$a_{w\,max} = \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\varphi} - 1\right] = \frac{mT}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1\right]$$

Minimum Number of Teeth Required to avoid interference

Let the adopted value of the addendum in some case be a_w times the module of teeth. Then this adopted value of the addendum must be less than the maximum value of the addendum to avoid interference.

i.e.
$$\frac{mT}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1\right] \geq a_w m$$

$$T \geq \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1}$$

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi}-1}$$

Interchangeable gears

Gears are interchangeable if they have:

- The same module
- The same pressure angle
- The same addendum and dedendum
- The same thickness

Non-Standard Gears

- Centre- distance Modifications

  - no. of teeth on a pinion can be reduced by increasing C-C distance and by changing marginally the tooth proportions and pressure angle.

  - Reduction in interference and improvement in contact ratio

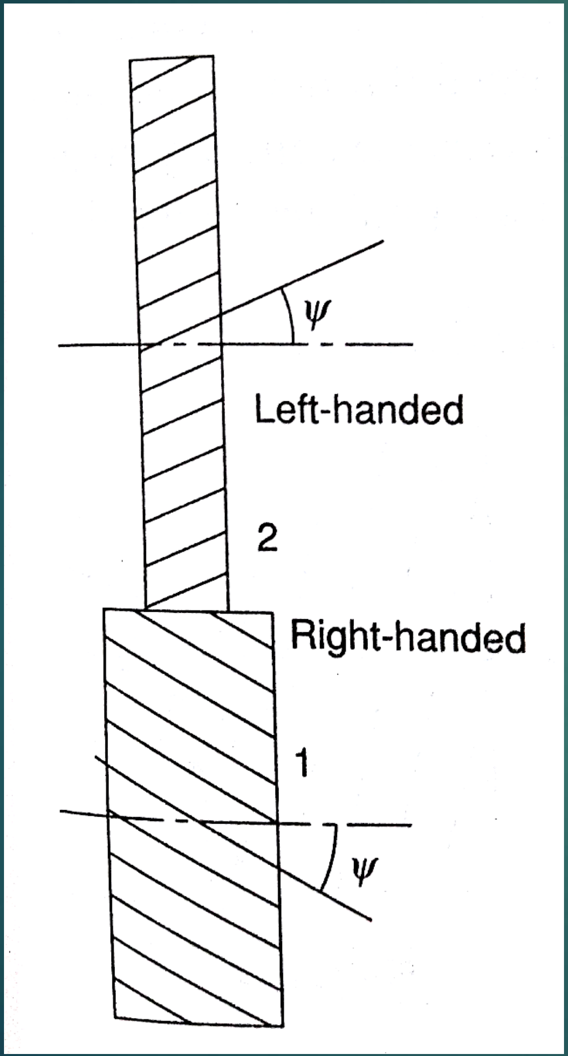
- Clearance Modifications

  - can be increased to 1.3 times module to 1.4 times module to have a larger fillet at the root of the tooth. As a result fatigue strength is increased

- Addendum modifications

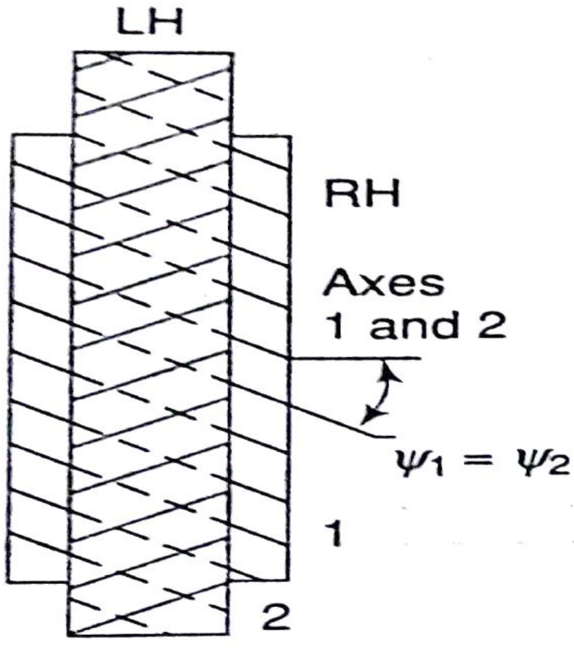
  - if this modification is carried out then there has to be no change in pitch circle radius and pressure angles

Helical and spiral gears

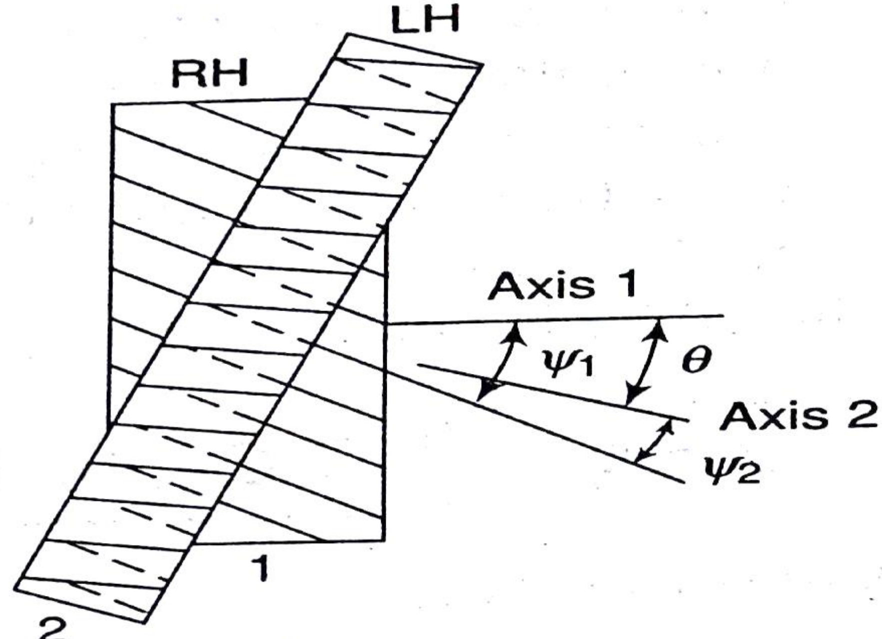


- Depending on the direction in which helix slopes away

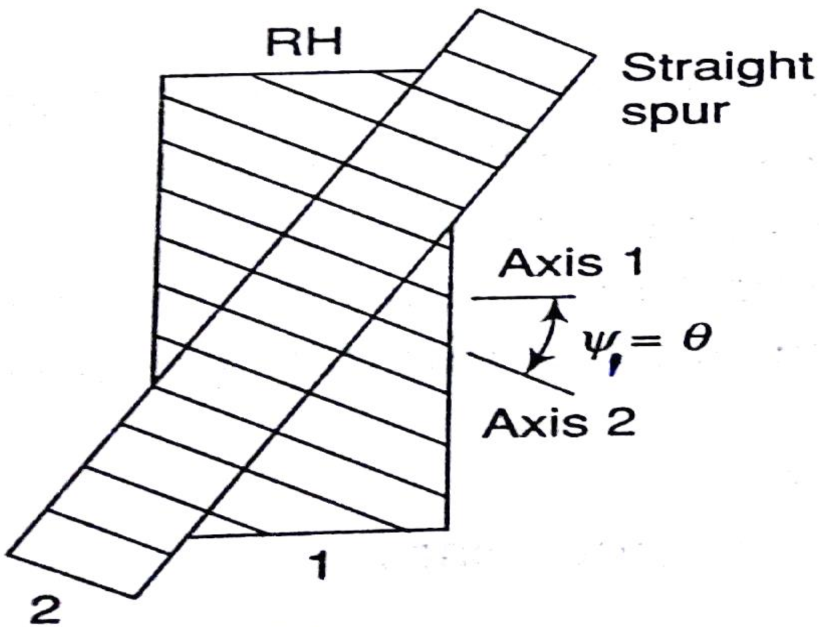
  1. Right handed
  2. Left handed


LH
RH
Axes
1 and 2
$\psi_1 = \psi_2$
1
2

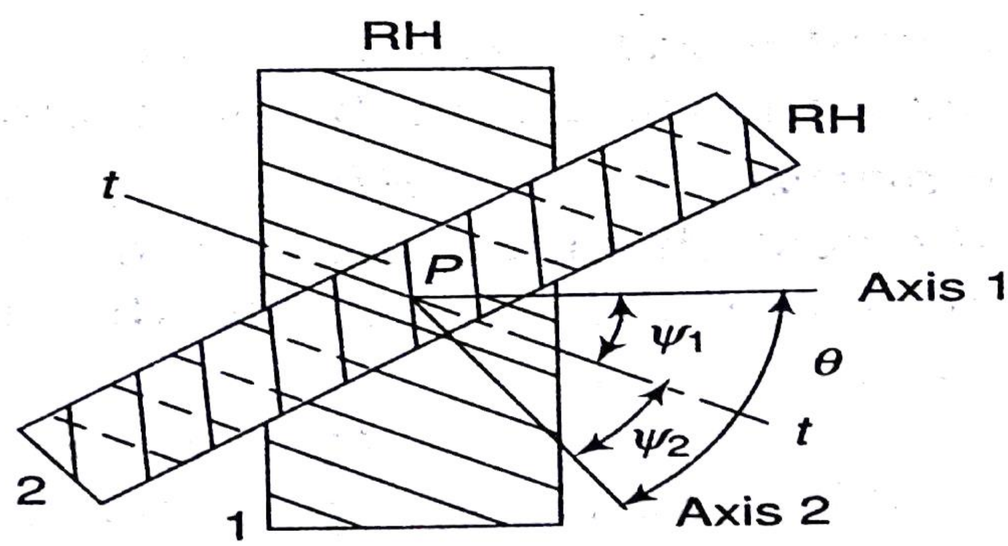
(a)


LH
RH
Axis 1
ψ_1
θ
Axis 2
ψ_2
1
2

(b)


RH
Straight
spur
Axis 1
$\psi_1 = \theta$
Axis 2
1
2

(c)


RH
RH
t
P
Axis 1
ψ_1
θ
t
ψ_2
Axis 2
2
1

(d)

Terminology of helical gears

Helix Angle (ψ) It is the angle at which the teeth are inclined to the axis of a gear. It is also known as *spiral angle*.

Circular Pitch (p) It is the distance between the corresponding points on adjacent teeth measured on the pitch circle. It is also known as *transverse circular pitch*.

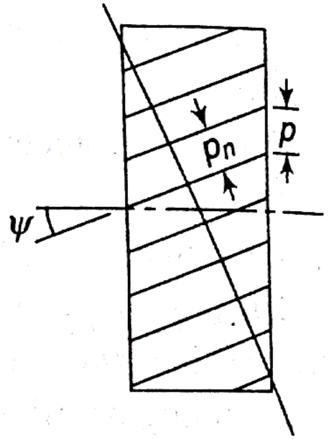
Normal Circular Pitch (p_n) Normal circular pitch or simply normal pitch is the shortest distance measured along the normal to the helix between corresponding points on the adjacent teeth. The normal circular pitch of two mating gears must be same.

$$P_n = p \cos \psi$$

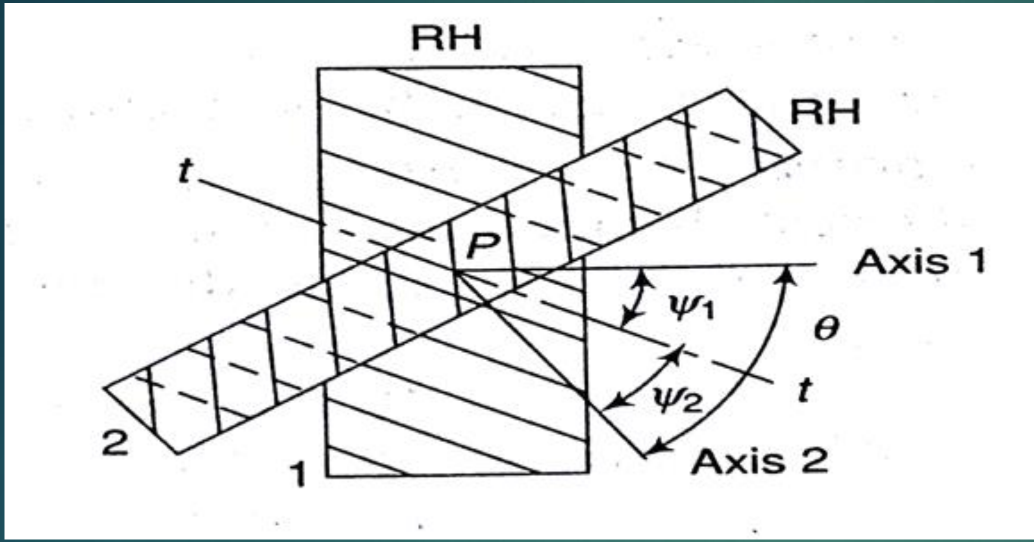
Also, we have, $p = \pi m$ as for spur gears

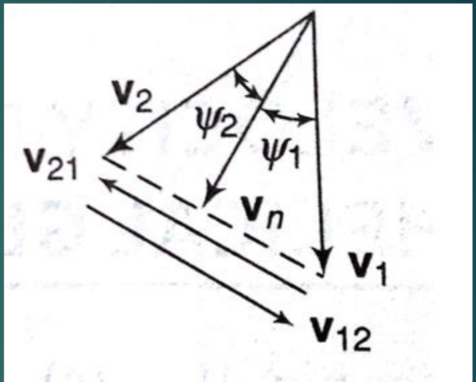
$$P_n = \pi m_n$$

and $m_n = m \cos \psi$



Velocity ratio of Helical gears





$$v_n = v_1 \cos\ \psi_1 = v_2 \cos\ \psi_2$$

$$v_n = v_1 \cos\psi_1 = v_2 \cos\psi_2$$

$$\frac{v_2}{v_1} = \frac{\cos\psi_1}{\cos\psi_2}$$

$$VR = \frac{\omega_2}{\omega_1} = \frac{v_2/r_2}{v_1/r_1} = \frac{v_2/d_2}{v_1/d_1} = \frac{d_1/v_2}{d_2/v_1}$$

$$VR = \frac{d_1}{d_2}\frac{\cos\psi_1}{\cos\psi_2} = \frac{m_1T_1}{m_2T_2}\frac{\cos\psi_1}{\cos\psi_2} = \frac{m_n/\cos\psi_1}{m_n/\cos\psi_2}\frac{T_1}{T_2}\frac{\cos\psi_1}{\cos\psi_2} = \frac{T_1}{T_2}$$

Centre to centre distance in helical gears

$$C = r_1 + r_2 = \frac{1}{2}(d_1 + d_1) = \frac{1}{2}(m_1 T_1 + m_2 T_2) = \frac{1}{2}\left(\frac{m_n}{\cos\psi_1}T_1 + \frac{m_n}{\cos\psi_2}T_2\right) = \frac{m_n}{2}\left(\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right)$$

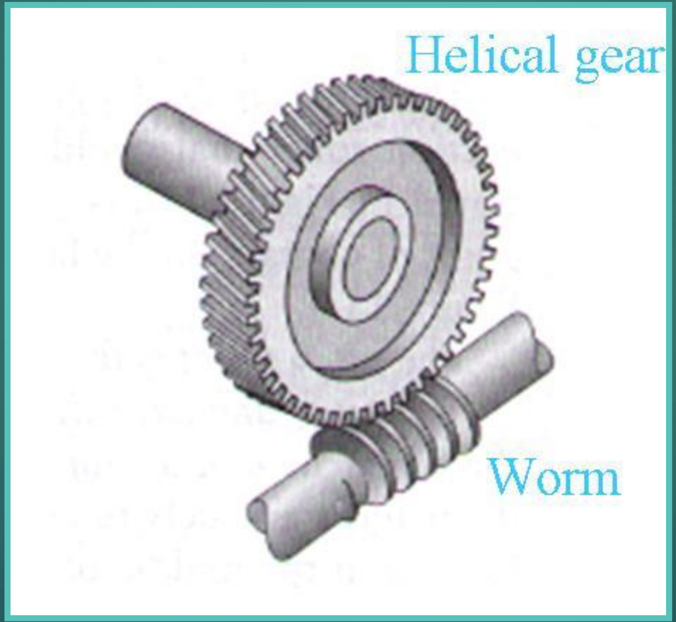
In case of helical gears of parallel shafts:

$$\psi_1 = \psi_2 = \psi$$

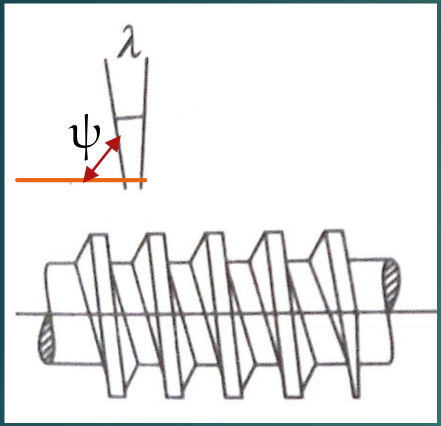
$$C = \frac{m_n}{2\cos\psi}(T_1 + T_2)$$

Worm and worm gear

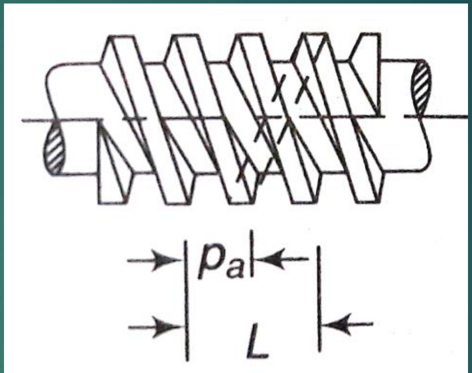
To transmit a higher load than usual spiral gears a worm and worm gear can be used. Large speed reduction can be also possible.



Worm and Worm Gear Terminologies



Single start



Double start

Axial Pitch (p_a) It is the distance between corresponding points on adjacent teeth measured along the direction of the axis.

Lead (L) The distance by which a helix advances along the axis of the gear for one turn around is known as lead.

In a single helix, the axial pitch is equal to lead. In a double helix, this is one-half the lead, in a triple helix, one third of lead, and so on.

Lead Angle (λ) It is the angle at which the teeth are inclined to the normal to the axis of rotation. Obviously, the lead angle is the complement of the helix angle.

$$\psi + \lambda = 90°$$

In case of worms, the lead angle is very small and the helix angle approaches 90^0.

As the shaft axis of the worm and worm gear are 90 deg.

$$\psi_1 + \psi_2 = 90°$$
$$(90° - \lambda_1) + \psi_2 = 90°$$
$$\lambda_1 = \psi_2$$

i.e., **lead angle of worm = helix angle of the gear wheel**

$$p_n \text{ of worm} = p_n \text{ of wheel}$$
$$p_{a1} \cos \lambda_1 = p_2 \cos \psi_2$$
$$\lambda_1 = \psi_2$$
$$p_{a1} = p_2$$

i.e., **Axial pitch of worm = Circular pitch of wheel**

Velocity Ratio

$$VR = \frac{\text{Angle turned by the gear}}{\text{Angle turned by the worm}}$$

Lead is also the distance turned by the pitch circle of the worm gear. There for angle turned by it during the same time will be

$$\frac{lead}{pitch\ circle\ radius\ of\ worm\ gear} = \frac{l}{r_2} = \frac{2l}{d_2}$$

Therefore velocity ratio is:

$$VR = \frac{\text{Angle turned by the gear}}{\text{Angle turned by the worm}} = \frac{2l / d_2}{2\pi} = \frac{l}{\pi d_2}$$

Centre to centre distance

$$C = \frac{m_n}{2}\left(\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right)$$

$$= \frac{m_2 \cos\psi_2}{2} \frac{1}{\cos\psi_2}\left(\frac{\cos\psi_2}{\cos\psi_1}.T_1 + T_2\right)$$

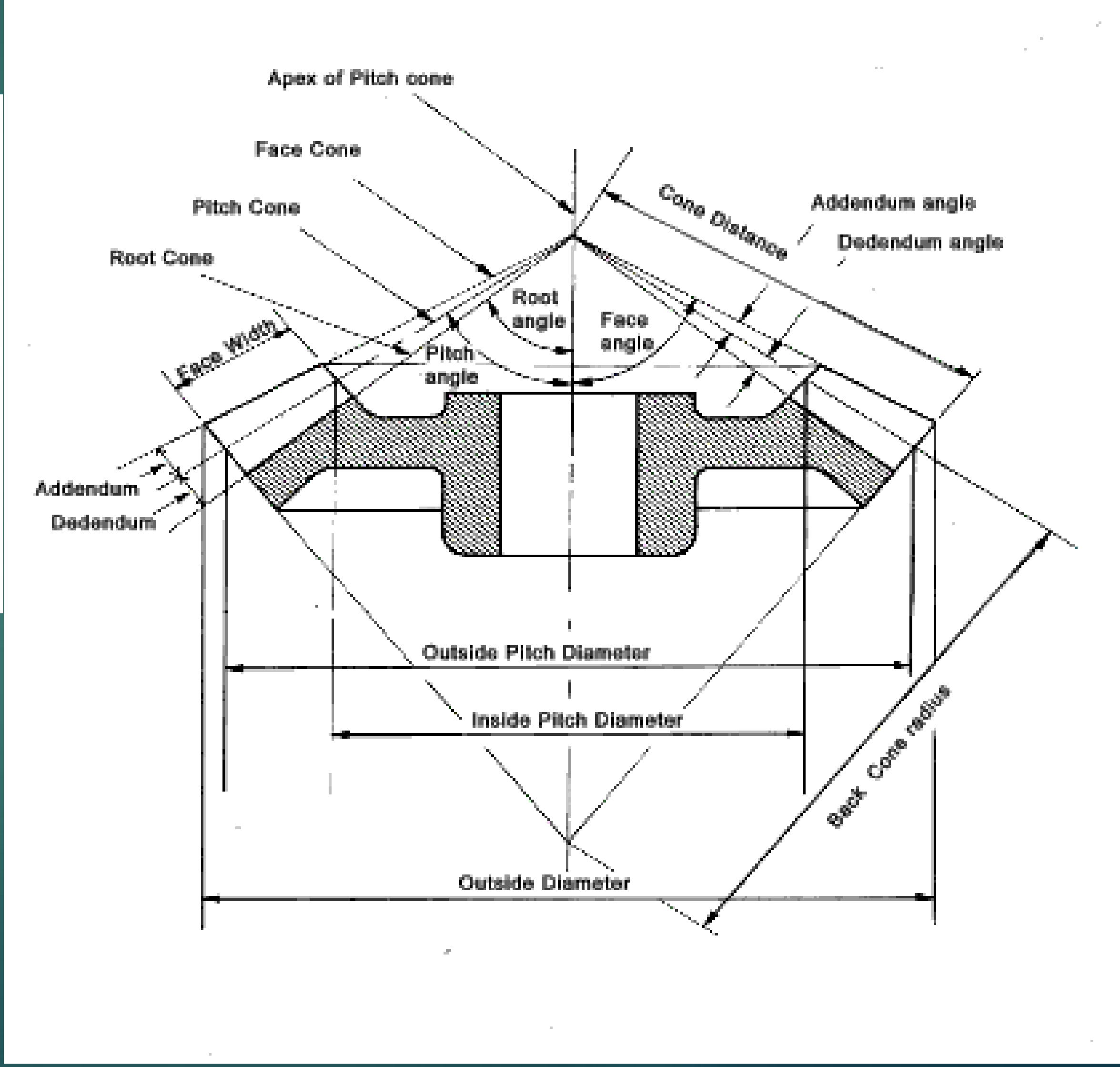
$$= \frac{m_2}{2}\left[\frac{\cos\lambda_1}{\cos(90^\circ - \lambda_1)} T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{\cos\lambda_1}{\sin\lambda_1} T_1 + T_2\right]$$

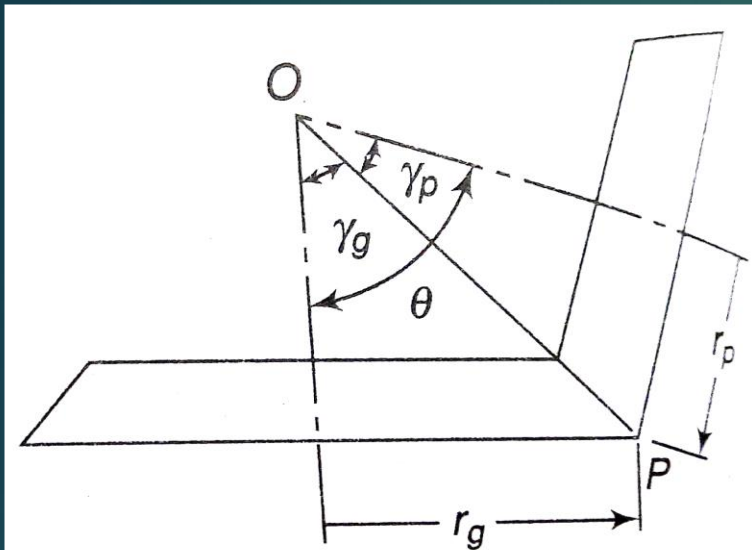
$$= \frac{m_2}{2}[T_1 \cot\lambda_1 + T_2]$$

$[\psi_2 = \lambda_1,\ \psi_1 = 90^\circ - \lambda_1]$

Bevel Gears



Apex of Pitch cone
Face Cone
Pitch Cone
Root Cone
Cone Distance
Addendum angle
Dedendum angle
Root angle
Face angle
Pitch angle
Face Width
Addendum
Dedendum
Outside Pitch Diameter
Inside Pitch Diameter
Outside Diameter
Back Cone radius



Let γ_g, γ_p = pitch angles of gear and pinion respectively

r_g, r_p = pitch radii of gear and pinion respectively.

$$\sin\gamma_g = \frac{r_g}{OP} = \frac{r_g}{r_p / \sin\gamma_p} = \frac{r_g}{r_p}\sin(\theta - \gamma_g)$$

$$\sin\gamma_g = \frac{r_g}{r_p}(\sin\theta\cos\gamma_g - \cos\theta\sin\gamma_g)$$

Dividing both sides by cos γ_g,

$$\tan\gamma_g = \frac{r_g}{r_p}(\sin\theta - \cos\theta\tan\gamma_g)$$

$$\frac{r_p}{r_g}\tan\gamma_g = \sin\theta - \cos\theta\tan\gamma_g$$

$$\tan \gamma_g = \frac{\sin \theta}{\frac{r_p}{r_g} + \cos \theta}$$

$$v_p = \omega_g r_g = \omega_p r_p \quad \text{or} \quad \frac{r_p}{r_g} = \frac{\omega_g}{\omega_p}$$

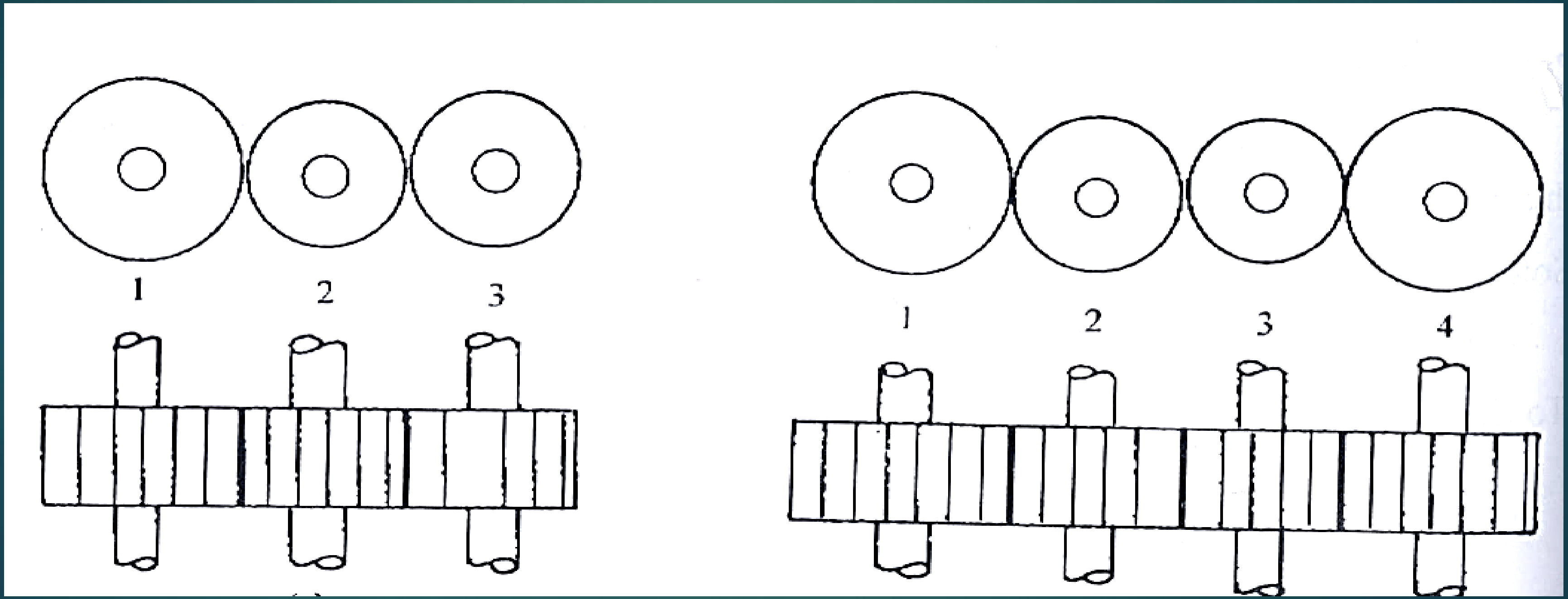
$$\tan \gamma_g = \frac{\sin \theta}{\frac{\omega_g}{\omega_p} + \cos \theta}$$

$$\tan \gamma_p = \frac{\sin \theta}{\frac{\omega_p}{\omega_g} + \cos \theta}$$

Gear trains

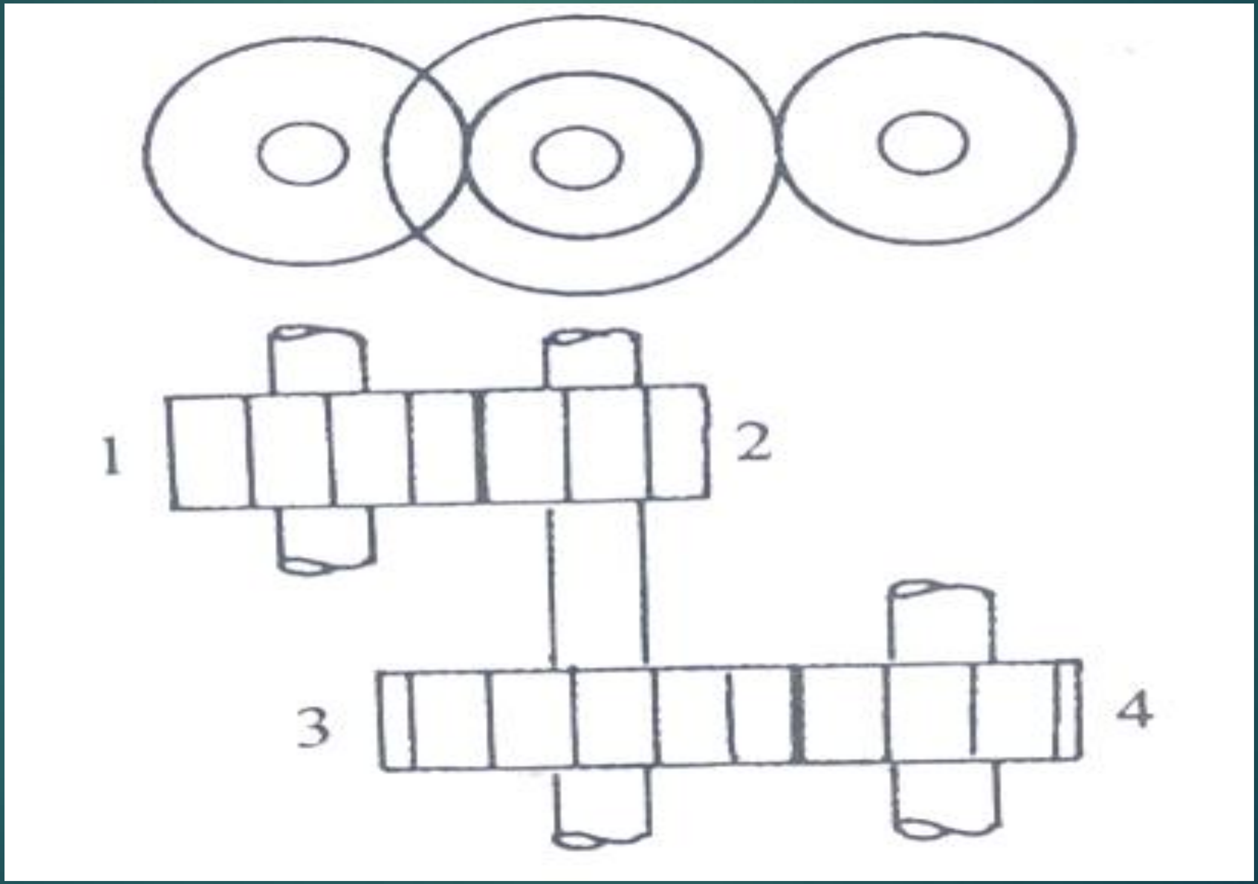
SIMPLE GEAR TRAIN

✓Each shafts carries only one gear
✓Intermediate gear have no effect on the velocity ration and hence known as idlers



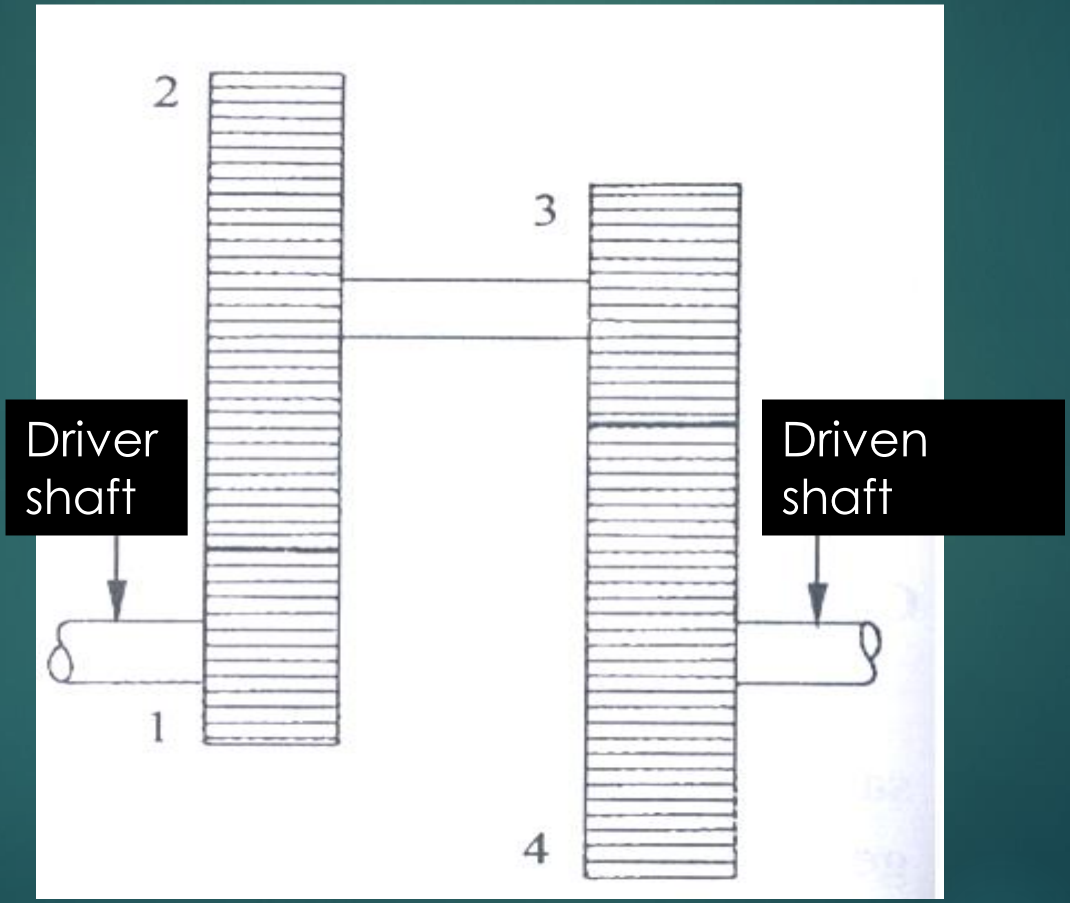
COMPOUND GEAR TRAIN

✓Gears are connected in such a way that two or more gears rotates about the same axis
✓Intermediate shafts carry more than one gear



REVERTED GEAR TRAIN

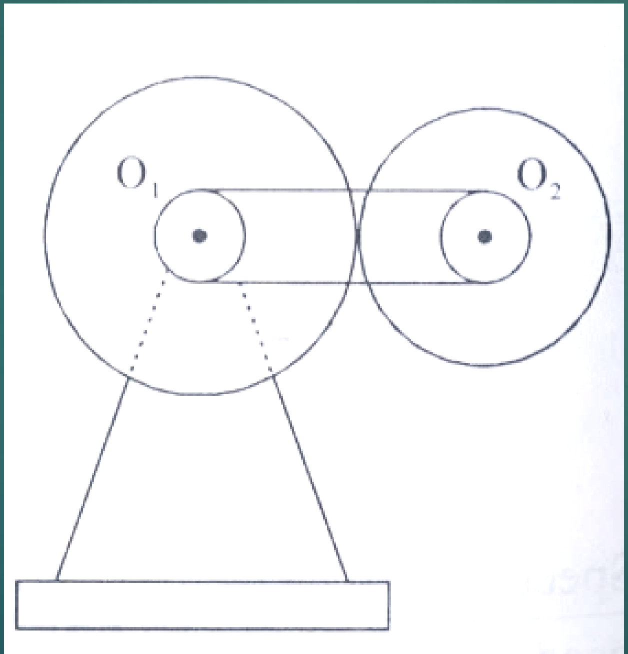
✓ here the axes of first and last gears coincide



EPICYCLIC GEAR TRAIN

✓Here the axis of rotation of one of the gear rotates about the fixed axis of rotation of another gear

Eg: Sun and planet gears
Differential mechanism of vehicles



Torques in epicyclic trains

Let N_S, N_a, N_P and N_A be the speeds and $\mathbf{T}_S$, $\mathbf{T}_a$, $\mathbf{T}_P$ and $\mathbf{T}_A$ the torques transmitted by S, a, P and A respectively

We have,

0

$$\mathbf{T} = 0$$

or

$$\mathbf{T}_S + \mathbf{T}_a + \mathbf{T}_P + \mathbf{T}_A = 0$$

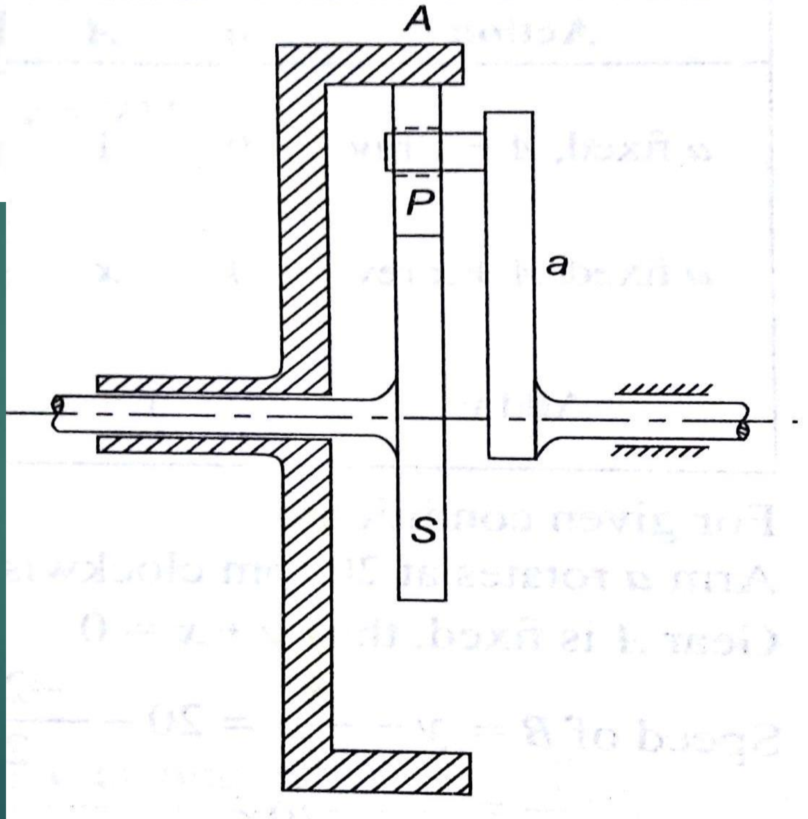
Theoretically we can say input work is equal to output work if no losses during transmission is considered, Then we can write,

$$\Sigma \mathbf{T} N = 0$$

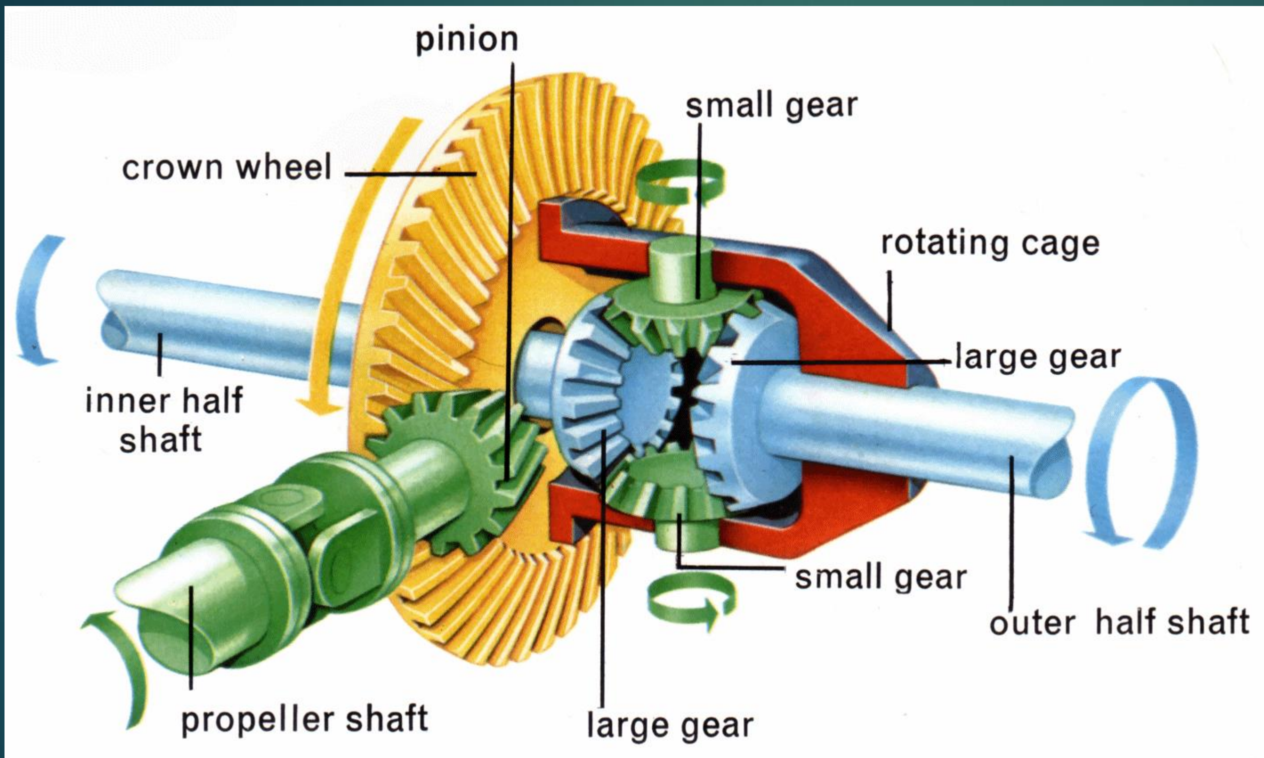
$$\mathbf{T}_S N_S + \mathbf{T}_a N_a + \mathbf{T}_A N_A = 0$$

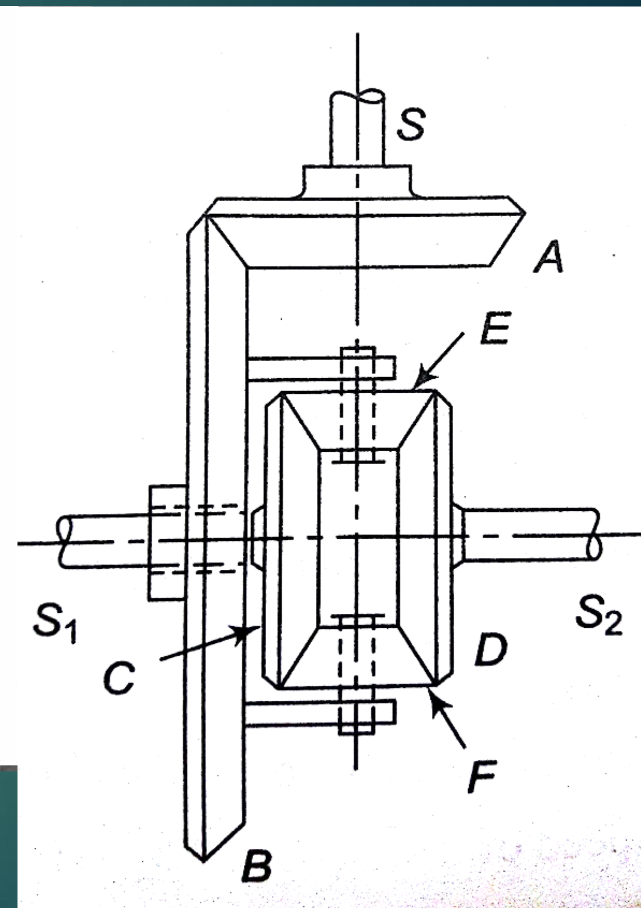
$$\text{If } A \text{ is fixed, } N_A = 0$$

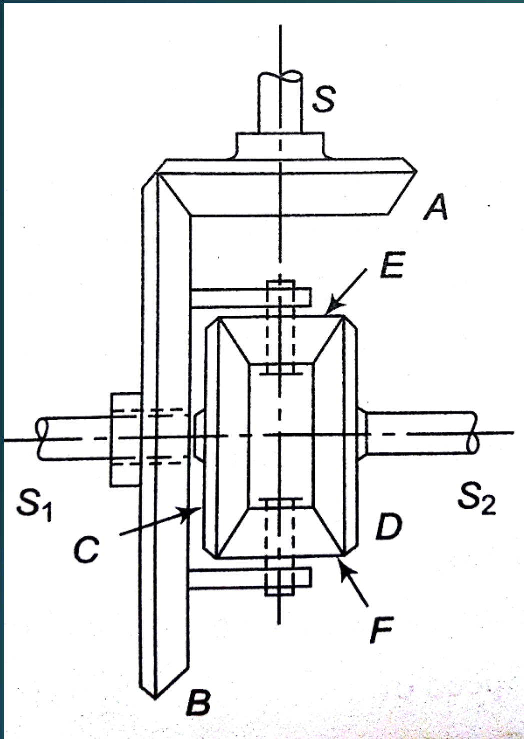
$$\mathbf{T}_S N_S + \mathbf{T}_a N_a = 0$$



Differentials







Action	B	C/S_1	D/S_2	E/F
B fixed, C + 1 rev.	0	1	–1	$\frac{T_C}{T_E}$
B fixed, C + x rev.	0	x	$-x$	$\frac{T_C}{T_E}x$
Add y	y	$y+x$	$y-x$	$\frac{T_C}{T_E}x+y$

From the above table, it can be observed that the speed of B is the arithmetical mean of the speeds of C and D, because $y = \{(y + x) + (y - x)\}/2$. This shows that while taking a turn, if the speed of C decreases than that of B, there will be a corresponding increase in the speed of D.

Automobile Transmission gear trains

Sliding gear box

- Make use of a compound gear train and is engaged by sliding the gears on the driven shaft to mesh with the gears on the lay shaft.
    I. Sliding mesh
    II. Constant mesh transmission

Pre selective gear box

- Make use of sun and planet gears