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Σ Sigma Mathematics

DEPARTMENT OF MATHEMATICS

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FROM THE HOD'S DESK

As a area of science, Mathematical Science has endless possibilities. Its interdisciplinary interaction with Computer Science, Management Science and Medical Sciences is indeed quite interesting as well as highly significant, proving it indespensable for today's advanced engineering and manufacturing demands. Ever since the department of Mathematics started its journey, the department has been simultaneously and successfully performing the multiple roles of creating new knowledge, acquiring new capabalities and producing an intelligent human resource pool contributing in various domains of the society. The department has alwaya been on a high growth path and has experienced and dedicated faculty with strong commitment to engineering education who work with enthusiasm to provide a vibrant and optimum learning environment.

MESSAGE FROM THE EDITORS

MATHEMATICS is the cradle of all creations

A number appreciated as early as 500 BC, earns its lofty name due to its aesthetical value: its unobtrusive and pervading presence is seen in architecture, paintings, music and nature. Sigma represent the totality which is the significance of Mathematics. Mathematics, the queen of sciences, is complete, which is understandable only when we observe the inner meaning of equations and formula.

That is the reason for choosing the name Sigma for this journal. The first issue contains several interesting articles not only to the mathematics community but also to a wider range of knowledge seekers. We thank all contributors for giving articles to this first issue.

Mathematics is the tool in our hand to make our life simpler and easier, to reach our target, to win our battle and to live safe. Realize and appreciate the beauty of the subject and embrace it with all our heart.

The editors of Sigma are Dr Binu R, Dr Ramkumar P B, P R Neethu and Selvy, Department of Mathematics, RSET.

THE CURVELET TRANSFORM

Dr Ramkumar P.B, Professor, Department of Mathematics, RSET

Applications of wavelets have become increasingly popular in scientific and engineering fields. The development of wavelets and related ideas led to convenient methods to reach into large data sets thereby to identify crucial properties in such data sets. In 1999, an anisotropic geometric wavelet transforms, named ridgelet transform, was proposed by Candes and Donoho. The straight-line singularities are optimally represented by this transform. But to analyze local line or curve singularities, the ridgelet transform have to apply on blocks. This block ridgelet transform is known as curvelet transform.

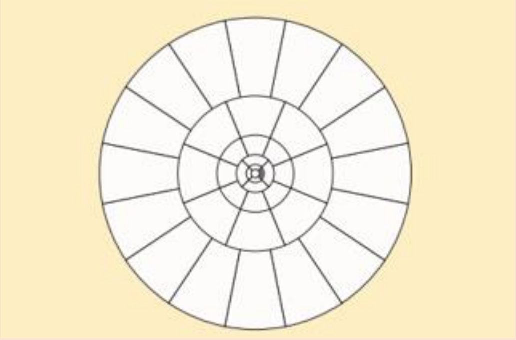
Wavelets are the generalization of Fourier transform which is represented in terms of a basis containing both location and spatial frequency. Directional wavelet transforms are using basis functions which are localized in orientation. A curvelet transform differ in this localized orientation. It also varies with scale. Basis functions are long ridges at scale j and its shape is 2^{j} by $2^{-j/2}$.Therefore, scale bases are skinny ridges with an orientation.

Signals and images exhibit discontinuities across curves. i.e line- like edges. This is known as line or curve singularities. Wavelets perform only at representing point singularities. It cannot perform at geometric properties of structures like irregularity of edges. So, wavelet-based structure feature extraction become computationally inefficient which contain geometric features with line and surface singularities. Curvelet transforms is defined by considering such type of singularities also.

Curvelet transform is useful to apply in multi scale object representation, image processing, scientific computing, fluid mechanics etc.

The one-dimensional wavelet transforms corresponding to a family of dilated and translated functions $\{\psi_{j,k} = 2^{\frac{j}{2}}\psi(2^{\,j}.-k): j.k\epsilon Z\}$, generated by one mother wavelet $\psi\epsilon L^{2}(R)$. Then any function $f\epsilon L^{2}(R)$ can be uniquely represented in wavelet expansion $f = \sum_{j,k} c_{j,k}(f)\,\psi_{j,k}$ where $c_{j,k}(f) = \langle f, \psi_{j,k}\rangle$ are the wavelet coefficients. Here the Fourier transformed elements of the wavelet basis have the form $\widehat{\psi_{j,k}}(\xi) = 2^{-\frac{j}{2}}e^{-i2^{-j}\xi k}\,\hat{\psi}\,(2^{-j}\xi)$, i.e dilation by 2^{j} in space domain corresponds to dilation by 2^{-j} in frequency domain, and the translation corresponds to a phase shift.

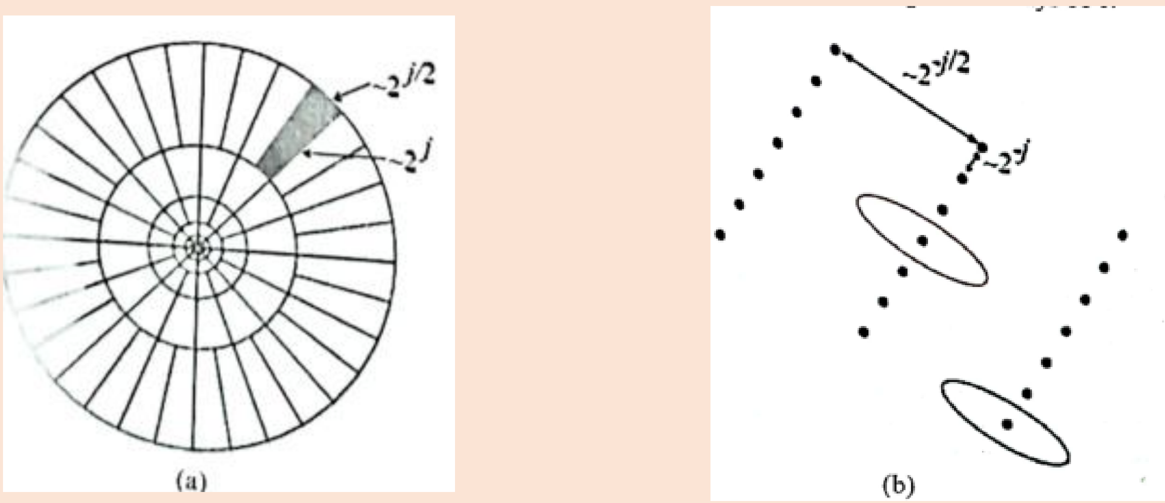
For curvelet is obtained by incorporating rotation invariance to the wavelets. So construct a frame using a basic curvelet ϕ with translation, dilation and rotation of ϕ. For the construction of curvelets, consider the polar coordinates in frequency domain and curvelet elements are supported near wedges. Refer the figure.



Definition: Let x be a special variable, ω a frequency domain variable, r and θ are polar coordinates in the frequency domain. Let $W(r)$ and $V(t)$ be the radial window function and angular window function with W taking positive real arguments and supported on $r\epsilon(\frac{1}{2},2)$ and V taking real arguments and supported on $t\epsilon[-1,1]$. $W(r)$ and $V(r)$ satisfies the following conditions: $\sum_{j=-\infty}^{\infty} W^{2}(2^{j}r) = 1, r \in (\frac{3}{4},\frac{3}{2})$, $\sum_{l=-\infty}^{\infty} V^{2}(t-l) = 1, t \in (\frac{-1}{2},\frac{1}{2})$

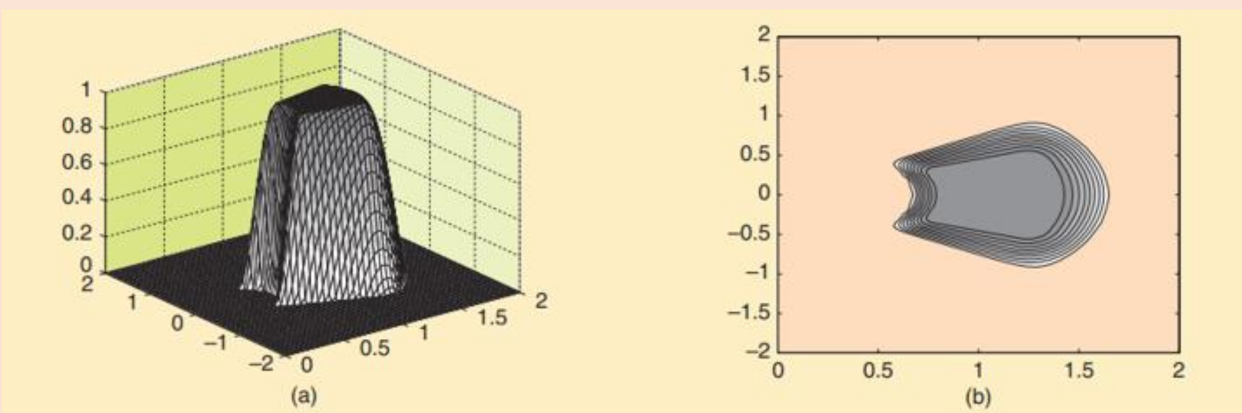
Let the frequency window U_j defined in terms of $W(r)$ and $U(t)$ be $U_j(r,\theta) = 2^{-\frac{3j}{4}}W(2^{-j}r)V(\frac{2^{[\frac{j}{2}]}\theta}{2\pi})$ where $[\frac{j}{2}]$ is the integer part of $\frac{j}{2}$. Hence the support of U_j is a polar wedge defined in terms of the support of W and V. Define $\widetilde{\phi_j}(\omega) = U_j(\omega), \widetilde{\phi_j}(\omega)$ tends to the Fourier transform of $\phi_j(x)$.

The curvelets at position $x_k^{j,l}$, at scale 2^{-j}, with orientation θ_l is defined as $\phi_{j,l,k}(x) = \phi_j\left(R_{\theta_l}(x - x_k^{j,l})\right)$, where $R_\theta$is the rotation by θ radians, $R_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \theta_l = 2\pi.2^{-[\frac{j}{2}]}l$ with $l = 0,1 \ldots$ such that , $0 \leq \theta_l \leq 2\pi$,the rotation angles,$k = (k_1, k_2) \in Z^2$, the translation parameters and $x_k^{j,l} = R_{\theta_l}^{-1}(k_1.2^{-j}, k_2.2^{-\frac{j}{2}})$.



Curvelets at frequency domain (a) and spatial domain (b) (Courtesy: Candes)

A curvelet coefficient is the inner product of $f \epsilon L^2(R^2)$ and a curvelet $\phi_{j,l,k}$ as $c_{j,l,k} = \langle f, \phi_{j,l,k} \rangle$.



Basic curvelet $\widetilde{\emptyset}_{\cdot 0,0,0}$in the (a) frequency domain and (b) its support.

Properties of Curvelet transform

1) Expansion of an arbitrary function: $f(x_1, x_2) \in L^2(R^2)$ as a series of curvelets $f = \sum_{j,l,k} \langle f, \phi_{j,l,k} \rangle$ with $\sum_{j,l,k} |\langle f, \phi_{j,l,k} \rangle|^2 = \|f\|^2, \forall f \in L^2(R^2)$.

2) Parabolic scaling: Effective length and width obey the anisotropic scaling relation, length$\approx 2^{-j/2}$, width$\approx 2^{-j}$ $\Rightarrow$width$\approx$length^2.
3) Oscillation: At scale 2^{-j}, a curvelet is a little needle whose envelope is a specified ridge of effective length $2^{-j/2}$ and width 2^{-j}. This describes an oscillatory behavior

4) Vanishing moments: The curvelet template $\emptyset_j$ is said to have q vanishing moments when $\int_{-\infty}^{\infty} \emptyset_j(x_1, x_2)\, x_1^n dx_1 = 0, \forall 0 \leq n < q, \forall x_2$.

Curvelets are an appropriate basis for representing images (or other functions) which are smooth apart from singularities along smooth curves, where the curves have bounded curvature, i.e. where objects in the image have a minimum length scale. This property holds for cartoons, geometrical diagrams, and text. As one zooms in on such images, the edges they contain appear increasingly straight. Curvelets take advantage of this property, by defining the higher resolution curvelets to be more elongated than the lower resolution curvelets. However, natural images (photographs) do not have this property; they have detail at every scale. Therefore, for natural images, it is preferable to use some sort of directional wavelet transform whose wavelets have the same aspect ratio at every scale.

When the image is of the right type, curvelets provide a representation that is considerably sparser than other wavelet transforms. This can be quantified by considering the best approximation of a geometrical test image that can be represented using only n wavelets, and analyzing the approximation error as a function of n. For a Fourier transform, the squared error decreases only as $\mathrm{O}(\frac{1}{\sqrt{\mathrm{n}}}\}$ For a wide variety of wavelet transforms, including both directional and non-directional variants, the squared error decreases as $\mathrm{O}(\frac{1}{\mathrm{n}})$. The extra assumption underlying the curvelet transform allows it to achieve $\mathrm{O}((\log \mathrm{n})^3/\mathrm{n}^2)$.

Efficient numerical algorithms exist for computing the curvelet transform of discrete data. The computational cost of the discrete curvelet transforms proposed by Candès et al. (Discrete curvelet transform based on unequally-spaced fast Fourier transforms and based on the wrapping of specially selected Fourier samples) is approximately 6–10 times that of an FFT, and has the same dependence of $\mathrm{O}(\mathrm{n}^2 \log \mathrm{n})$ for an image of size $n \times n$.

A NOTE ON NEUTROSOPHIC G-SUBMODULES

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Abstract. This paper discusses the basic notions and fundamental characteristics of neutrosophic G-submodules. The analysis and support of neutrosophic G-submodules, as well as the characteristics of any arbitrary family of neutrosophic G-submodules, are all included in this paper.

Neutrosophic G-submodules

Definition 1.1 "Let $(G,*)$ be a group. A vector space M over the field K is called a *G-module*, denoted as G_M, if for every $g \in G$ and $m \in M$; $\exists$ a product (called the action of G on M), $g \cdot m \in M$ satisfies the following axioms

1. $1_G \cdot m = m$; $\forall m \in M$ (1_G being the identity element of G)
2. $(g * h) \cdot m = g \cdot (h \cdot m)$; $\forall m \in M$ and $g, h \in G$
3. $g \cdot (k_1 m_1 + k_2 m_2) = k_1(g \cdot m_1) + k_2(g \cdot m_2)$; $\forall\, k_1, k_2 \in K$; $m_1, m_2 \in M$" .

Definition 1.2. "A *neutrosophic set* P of the universal set X is defined as $P = \{(x, t_P(x), i_P(x), f_P(x)) : x \in X\}$ where $t_P, i_P, f_P : X \to (^-0, 1^+)$. The three components t_P, i_P *and* f_P represent membership value (Percentage of truth), indeterminacy (Percentage of indeterminacy) and non membership value (Percentage of falsity) respectively. These components are functions of non standard unit interval $(^-0, 1^+)$ ".

Remark 1.1. "If $t_P, i_P, f_P : X \to [0,1]$, then P is known as single valued neutrosophic set (SVNS)"

Definition 1.3. Let M be a G module over a field K and $A \in U^{G_M}$, where U^{G_M} denotes the set of all neutrosophic set of M. Then the neutrosophic set $A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in M\}$ is said to be a neutrosophic G-module if the following conditions are satisfied:

1. $t_A(ax + by) \geq t_A(x) \wedge t_A(y)$

   $i_A(ax + by) \geq i_A(x) \wedge i_A(y)$

   $f_A(ax + by) \leq f_A(x) \vee f_A(y), \forall\, x, y \in M, a, b \in K$

2. $t_A(gm) \geq t_A(m)$

$i_A(gm) \geq i_A(m)$

$f_A(gm) \leq f_A(m)\ \forall\ g \in G, m \in M$

Definition 1.4. "Let M be a G-module. A vector subspace N of M is a G-*submodule* if N is also a G-module under the same action of G ".

Remark 1. 2. We denote the set of all neutrosophic G-submodules of M by $U(G_M)$.

Remark 1.3. Let A be a neutrosophic G-submodule of a G-module M, then it is obvious that $t_A(0) \geq t_A(m),\ i_A(0) \geq i_A(m)\ and\ f_A(0) \leq f_A(m)\ \forall\ m \in M$.

Theorem 1.4. If $A, B \in U(G_M)$, then $A \cap B \in U(G_M)$.

Definition 1.5. Let $A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in X\} \in U^{G_M}$. The support A^* of the neutrosophic G-submodule A can be defined as $A^* = \{x \in X, t_A(x) > 0, i_A(x) > 0, f_A(x) < 1, \forall x \in G_M\}$

Proposition 1.1. If $A \in U(G_M)$, then the support A^* is a G-submodule of M.

Remark 1.5. The converse of the above theorem need not be true as we see in the following example.

Example 1.1. Let $G = \{1, -1, i, -i\}$ be a group of 4^{th} root of unity under the binary operation 'multiplication'and $M = \mathbb{C}$ be the G-module over the field R. Define a neutrosophic set

$A = \{z, t_A(z), i_A(z), f_A(z): z \in M\}$ as follows

$$t_A(z) = \begin{cases} 1 & if\ z = 0 \\ 0.45 & if\ z \in R - \{0\}, \\ 0.30 & if\ z \in \mathbb{C} - R \end{cases} \quad i_A(x) = \begin{cases} 1 & if\ z = 0 \\ 0.50 & if\ z \in R - \{0\}, \\ 0.30 & if\ z \in \mathbb{C} - R \end{cases}$$

$$f_A(x) = \begin{cases} 0 & if\ z = 0 \\ 0.25 & if\ z \in R - \{0\} \\ 0.50 & if\ z \in \mathbb{C} - R \end{cases}$$

Here the support $A^* = M$ which is a G-module. But A is not a neutrosophic G-submodule. Take $m = 2 \in M\ and\ g = i$, then $t_A(gm) = t_A(2i) = 0.30 \ngeq 0.45 = t_A(2) = t_A(m)$. Similarly, $i_A(gm) \ngeq i_A(m)\ and\ f_A(gm) \nleq f_A(m)$. Therefore $A \notin U(G_M)$.

Definition 1.6. If for all $\beta \in [0,1]$, the β-level sets of a neutrosophic G-module A, can be denoted and defined as $A_\beta = \{x \in M: t_A(x) \geq \beta, i_A(x) \geq \beta, f_A(x) \leq \beta\}$.

Proposition 1.2. If $A \in U(G_M)$, then the β level set A_β of A is a G-submodule of M.

Proposition 1.3. Let $A,\ B \in U^{G_M}$. If $A \subseteq B$, then $A^* \subseteq B^*$.

Proposition 1.4. Let $A_i,\ i \in J$ be an arbitrary non empty family of U^{G_M}. Then for any $\beta \in [0,1]$, then

1. $\bigcap_{i\in J}(A_i)_\beta = \left(\bigcap_{i\in J} A_i\right)_\beta$
2. $\bigcup_{i\in J}(A_i)_\beta \subseteq \left(\bigcup_{i\in J} A_i\right)_\beta$

Theorem 1.6. Let $A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in M\}$ be a neutrosophic G submodule of M over a field K. Then for each $r \in [0,1]$, the neutrosophic set $A_r = \{\left(x, t_{A_r}(x), i_{A_r}(x), f_{A_r}(x)\right) : x \in M\}$ defined by $t_{A_r}(x) = t_A(x) \wedge r,\ i_{A_r}(x) = i_A(x) \wedge r,\ f_{A_r}(x) = f_A(x) \vee (1-r)\ \forall\ x \in M$ is a neutrosophic G submodule.

Theorem 1.7. Let $A_i,\ i \in J$ be an arbitrary non empty family of $U(G_M)$, then $\bigcap_{i\in J} A_i \in U(G_M)$.

Theorem 1.8. Let $A \in U(G_M)$. Then P hold the following:

1. $rA \subseteq A\ \forall r \in R$
2. $A + A \subseteq A$

Conclusion

The neutrosophic G-submodules are extensively studied in this work, with an emphasis on the characteristics and properties. As algebraic structures that combine neutrosophic sets with group actions, neutrosophic G-submodules are a special and significant matter of study in algebraic representation theory. this work derived the essential characteristics of neutrosophic G-submodules and establishing their mathematical framework and formal definition. It studies how they behave in various algebraic situations, the degree to which they are under group operations, and how they relate to other mathematical structures. Exact Sequences and category theory in neutrosophic G-modules are the next levels to explore.

VEDIC MATHEMATICS

Bindu V A, Assistant Professor, Department of Mathematics, RSET

A name so synonymous with Ancient India. Ancient India was adept in the system of Mathematics that pervaded their lives and enabled them to gain the most of what Mother Earth had to offer them. They lived in synergy with nature with an assimilation of knowledge that both bewilders and amazes us.

Of late, there has been a growing interest in the Vedic system of Mathematics which are found to answer difficult and sought after problems. This has renewed multifaceted research towards this stream of mathematics. The earliest recorded Vedic period or the Vedic age is presumed to be 1500 to 500 BCE; which surprisingly coincides with the later period of the Bronze age and the early Iron age. It was also during this period that Vedas and Puranas came to guide man in various aspects of life. Vedas are thought to be created by Brahma, but in essence it is a composed writing of various Rishis (sages) putting together their experiences and life journeys. The Puranas were composed by Maharishi Vyasa. With this introduction to the Vedic period, let's try to understand where Vedic Mathematics originated from and how it permeated the lives of people during those times.

In recent times, the knowledge of Vedic mathematics has been revitalized by Sri Bharati Tirthaji (1884-1960). With his research and knowledge gained over his life; he put together some potent information for Vedic mathematics. According to the base knowledge of Vedic Mathematics, it is composed on sixteen SUTRAS (word formulae). The SUTRAS are designed in such a way that it traces the way how the human brain works and this method enables a student to trace and learn these solutions in the most natural manner. Once the student has gained the required knowledge, they can then move to the appropriate method of solution solving. The most notable aspect is that of coherence. For example, a technique will guide the student to carry out large multiplications but with wishing thinking and application, they can also reverse the technique to do divisions, squaring and square roots. The idea is to practice and gain mastery over these techniques.

As mentioned earlier, the 16 SUTRAS of Vedic Mathematics easily aid in the study of mathematics. These are

Vedic Mathematics Sutras	Meaning
Ekadhikina Purvena	By one more than the previous one
Nikhilam Navatashcaramam Dashatah	All from 9 and the last from 10
Urdhva-Tiryagbyham	Vertically and crosswise
Paraavartya Yojayet	Transpose and adjust
Shunyam Saamyasamuccaye	When the sum is the same, that sum is zero
(Anurupye) Shunyamanyat	If one is in ratio, the other is zero
Sankalana-vyavakalanabhyam	By addition and by subtraction
Puranapuranabyham	By the completion or non-completion

Chalana-Kalanabyham	Differences and Similarities
Yaavadunam	Whatever the extent of its deficiency
Vyashtisamanstih	Part and Whole
Shesanyankena Charamena	The remainders by the last digit
Sopaantyadvayamantyam	The ultimate and twice the penultimate
Ekanyunena Purvena	By one less than the previous one
Gunitasamuchyah	The product of the sum is equal to the sum of the product
Gunakasamuchyah	The factors of the sum is equal to the sum of the factors

You will notice the meaning itself conveys the method to be used for calculation and hence enables the direction to solve the problems.

Benefits in understanding Vedic Mathematics

As you become proficient in the usage of Vedic Mathematics skills and techniques, there are huge benefits to reap and inculcate into our system of learning. A few of them are highlighted below:

- Solving problems faster,
- Avoids silly errors,
- Able to solve tricky equations,
- Reduces level of memorization,
- Increases focus of students to learn more,
- Helps in logical reasoning skills, and
- Fortifies the basic mathematical skills.

With a little more application and focused learning, we will be able to assimilate the skills of Vedic Mathematics in our being and become adept in one of the most ancient skills of learning and problem solving.

SOME FUZZY GRAPHS

Abraham Jacob, Assistant Professor, Department of Mathematics, RSET

The origin of graph theory started with the problem of Konigsberg bridge, in 1735. This problem leads to the concept of Eulerian Graph. The concept of tree was implemented by Kirchhoff, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. Then, Kirkman and Hamilton studied cycles on polyhedral and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a Fuzzy Graph Model. Most of our traditional tools for formal modeling, reasoning, and computing are crisp, deterministic and precise in character. In set theory, an element can either belong to a set or not; and in optimization, a solution is either feasible or not. So, for a tie situation the cases are not perfectly placed. Fuzzy set theory gives an idea for solving this type of problem. The theory of fuzzy set was proposed by Zadeh to handle the various uncertainties in many real applications. The theory of fuzzy sets is, basically, a theory of graded concepts- a theory in which everything is a matter of degree or, to put it figuratively, everything has elasticity. In the two decades since its inception, the theory has matured into a collection of concepts and techniques for dealing with complex phenomena that do not lend themselves to analysis by classical methods. Since complete information in science and technology is not always available, we need some other idea to solve those types of problems. After that Rosenfeld introduced fuzzy graphs, though Yeh and Bang also introduced this independently. Fuzzy graph is useful to represent some special relationship which is related with uncertainty.

A fuzzy graph $G = (V, \mu, \gamma)$ is a nonempty set V together with a pair of functions $\mu: V \to [0,1]$ and $\gamma: V \times V \to [0,1]$ such that for all $v_i, v_j \in V$, $\gamma(x, y) \le \mu(v_i) \wedge \mu(v_J)$. We call μ the fuzzy vertex set of G and γ the fuzzy edge set of G, respectively.

Atanassov introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFG). Theory of intuitionistic fuzzy sets (IFSs) creates an exponential growth in Mathematics and its applications. This ranges from traditional Mathematics to Information Sciences. This influences us to consider IFGs and their applications.

An Intuitionistic fuzzy graph (IFG) is of the form $G = (G^1, G^2, \mu_1, \gamma_1, \mu_2, \gamma_2)$, where

1. $G^1 = \{v_1, v_2 \dots v_n\}$ such that $\mu_1: G^1 \to [0,1]$ and $\gamma_1: G^1 \to [0,1]$,

the membership function and non membership function of the element $v_i \in G^1$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in G^1$, $i = 1,2, \dots n$.

2. $G^2 \subseteq G^1 \times G^1$ where $\mu_2: G^2 \to [0,1]$ and $\gamma_2: G^2 \to [0,1]$ are such that

   (a) $\mu_2(e_{v_i v_j}) \le \min\left(\mu_1(v_i), \mu_1(v_j)\right)$

   (b) $\gamma_2(e_{v_i v_j}) \le \max\left(\gamma_1(v_i), \gamma_1(v_j)\right)$

   (c) $0 \le \mu_2(e_{v_i v_j}) + \gamma_2(e_{v_i v_j}) \le 1$ for every edges $e_{v_i v_j} \in G^2$,

   $i = 1,2, \dots n, j = 1,2, \dots n$.

IFG can be used to solve some real-life problems. Let us consider set V = {A, B, C, D} of four different products A, B, C and D available in market. The chances of rising of price and not rising of price are considered as the membership and non-membership values of each product(vertex), and edge between each pair of vertices represents Dipesh Chakraborty and Nirmal Kumar Mahapatra 22 the relation of price rising [that is, a relative change of price of one item with respect to other] with some membership and non-membership values. It is defined that a product is said to be most valuable (and hence its market value is stable) when the difference between membership value and non-membership value is minimum among all the products. So, a businessman may purchase most valuable products for his / her smooth business.

In fuzzy graph, each vertex and edge have membership functions which lie in the interval [0,1] and sum of membership function and its complement of each vertex and edge is unity, whereas in IFG, sum of membership function and non-membership function of each vertex and edges in IFG should lie in the interval [0,1]. However, IFG cannot handle the situations when this sum is greater than unity. In particular, if membership function of a vertex is 0.6 and its non-membership function is 0.5, then their sum=0.6+0.5=1.1 which is greater than one. Such problems cannot handle with IFG.

Yagar introduced Pythagorean fuzzy sets to handle uncertainty situations. As a continuation of this, Rajkumar defined Pythagorean fuzzy graphs (PFG).
A Pythagorean fuzzy graph (PFG) is of the form
$G = (G^1, G^2, \mu_1, \gamma_1, \mu_2, \gamma_2)$, where

1. $G^1 = \{v_1, v_2 \dots v_n\}$ such that $\mu_1 : G^1 \to [0,1]$ and $\gamma_1 : G^1 \to [0,1]$,

the membership function and non membership function of the element $v_i \in G^1$ respectively and $0 \le [\mu_1(v_i)]^2 + [\gamma_1(v_i)]^2 \le 1$ for every $v_i \in G^1$, $i = 1,2, \dots n$.

2. $G^2 \subseteq G^1 \times G^1$ where $\mu_2 : G^2 \to [0,1]$ and $\gamma_2 : G^2 \to [0,1]$ are such that
   (a) $\mu_2(e_{v_i v_j}) \le \min(\mu_1(v_i), \mu_1(v_j))$
   (b) $\gamma_2(e_{v_i v_j}) \le \max(\gamma_1(v_i), \gamma_1(v_j))$
   (c) $0 \le \left[\mu_2(e_{v_i v_j})\right]^2 + \left[\gamma_2(e_{v_i v_j})\right]^2 \le 1$ for every edges $e_{v_i v_j} \in G^2$,
   $i = 1,2, \dots n, j = 1,2, \dots n$.

In PFG, sum of squares of membership function and non-membership function of each vertices and edges in PFG should lie in the interval [0,1]. The main advantage of this membership functions is that this involves all Intuitionistic fuzzy membership functions.

FUZZY LOGIC CONTROLLER DESIGN FOR GUN-TURRET SYSTEM

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Fuzzy logic is a form of many-valued logic. It deals with reasoning that is approximate rather than fixed and exact. Compared to traditional binary sets (where variables may take on true or false values) fuzzy logic variables may have a truth value that ranges in degree between 0 and 1. The term "fuzzy logic" was introduced with the 1965 proposal of fuzzy set theory by Lotfi A. Zadeh. Fuzzy logic has been applied to many fields, from control theory to artificial intelligence and it is a form of knowledge representation suitable for notions that cannot be defined precisely, but which depend upon their contexts.

To illustrate some basic concepts in Fuzzy Logic, consider an example of a thermostat controlling a heater fan. The room temperature detected through a sensor is input to a controller which outputs a control force to adjust the heater fan speed. A conventional thermostat works like an on-off switch. If we set it at 78^{o}F then the heater is activated only when the temperature falls below 75°F. When it reaches 81oF the heater is turned off. As a result, the desired room temperature is either too warm or too hot. A fuzzy thermostat works in shades of grey where the temperature is treated as a series of overlapping ranges. For example, 78^{o}F is 60% warm and 20% hot. The controller is programmed with simple if-then rules that tell the heater fan how fast to run. As a result, when the temperature changes the fan speed will continuously adjust to keep the temperature at the desired level.

First step in designing such a fuzzy controller is to characterize the range of values for the input and output variables of the controller. Then assign labels such as cool for the temperature and high for the fan speed, and write a set of simple English-like rules to control the system. Inside the controller all temperature regulating actions will be based on how the current room temperature falls into these ranges and the rules describing the system behaviour. The controller's output will vary continuously to adjust the fan speed.

The temperature controller described above can be defined in four simple rules:

IF temperature IS cold **THEN** fan speed IS high

IF temperature IS cool **THEN** fan speed IS medium

IF temperature IS warm **THEN** fan speed IS low

IF temperature IS hot **THEN** fan speed IS zero

Here the linguistic variables cool, warm, high, etc. are labels which refer to the set of overlapping values. These triangular shaped values are called membership functions.

A fuzzy controller works similar to a conventional system: it accepts an input value, performs some calculations, and generates an output value. This process is called the Fuzzy Inference Process and works in three steps:

(a) Fuzzification where a crisp input is translated into a fuzzy value,

(b) Rule Evaluation, where the fuzzy output truth values are computed, and

(c) Defuzzification where the fuzzy output is translated to a crisp value.

During the fuzzification step the crisp temperature value of 78^{o}F is input and translated into fuzzy truth values. For this example, 78^{o}F is fuzzified into warm with truth value 0.6 (or 60%) and hot with truth value 0.2 (or 20%).

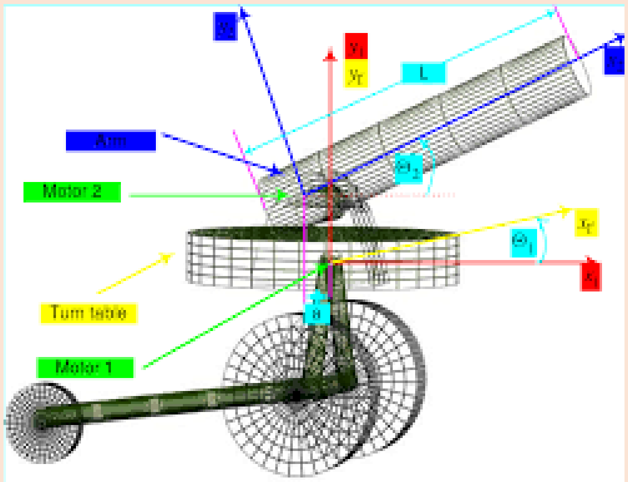
During the rule evaluation step the entire set of rules is evaluated and some rules may fire up. For 78oF only the last two of the four rules will fire. Specifically, using rule three the fan speed will be low with degree of truth 0.6. Similarly, using rule four the fan speed will be zero with degree of truth 0.2.
During the defuzzification step the 60% low and 20% zero labels are combined using a calculation method called the Centre of Gravity (COG) in order to produce the crisp output value of 13.5 RPM for the fan speed.

Fuzzy Logic in Control Systems.

Fuzzy Logic provides a more efficient and resourceful way to solve Control Systems. High precision control is desirable for future weapon systems. Several control design methodologies are applied to a weapon system to assess the applicability of each control design method and to characterize the achievable performance of the gun-turret system in precision controlFuzzy logic control (FLC) provides an effective means of capturing the approximate, inexact nature of the real world, and to address unexpected parameter variations and anomalies. Viewed in this perspective, the essential part of the FLC is a set of linguistic control rules related by the dual concepts of fuzzy implication and the compositional rule of inference. In essence, the FLC provides an algorithm which can convert the linguistic control strategy based on expert knowledge into an automatic control strategy.

Fuzzy logic Controller Design for gun-turret system.

The design objective of the gun-turret control system is to achieve a rapid and precise tracking response with respect to the turret motor command from the fire control system under the influences of disturbances, nonlinearities, and modeling uncertainties.



A fuzzy scheme is proposed for control of multi-body, multi-input and multi- output nonlinear systems with joints represented by a gun turret-barrel model which consists of two subsystems: two motors driving two loads (turret and barrel) coupled by nonlinear dynamics. Fuzzy control schemes are employed for compensation and nonlinear feedback control laws are used for control of nonlinear dynamics.

LAGRANGIAN POINT

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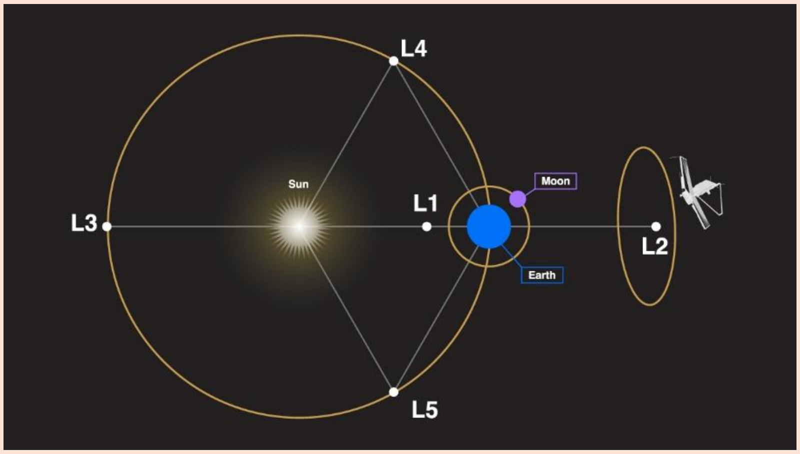
A Lagrangian point (also known as L point or Lagrange point or Libration point) is defined as the point where the force exerted between two large bodies in orbit becomes equal. For every combination of two large orbital bodies, there are five Lagrangian points, namely L_1, L_2, L_3, L_4 and L_5.

Of the five Lagrange points, L_1, L_2 and L_3 are unstable Lagrange points and they lie along the line connecting the two large masses whereasL_4 and L_5 are stable Lagrange points and they form the apex of two equilateral triangles that have large masses at their vertices.

Lagrange points are widely used in astronomy and one of the applications is that satellites are located at the Lagrange points in the Earth-Sun system.



The mathematical representation of each of the Lagrange points are as follows. Let R be the distance between the two main objects and M_1 and M_2 be the masses of the large and small objects, respectively.

L_1 is the point that lies between two large masses M_1 and M_2 and on the line defined by them. The gravitational attraction of M_1 is partially cancelled by the gravitational force of M_2. If r is the distance of an L_1 point from the smaller object, then

$$\frac{M_1}{(R-r)^2} = \frac{M_2}{r^2} + \frac{M_1}{R^2} - \frac{r(M_1+M_2)}{R^3}$$

L_2 is the point that lies on the line defined by the two large masses M_1 and M_2 and beyond M_2. The centrifugal effect on a body at L_2 is balanced by the gravitational force of the two large masses. Let r be the distance of L_2 point from M_2, then

$$\frac{M_1}{(R+r)^2} + \frac{M_2}{r^2} = \frac{M_1}{R^2} + \frac{r(M_1+M_2)}{R^3}$$

L_3 is the point that lies on the line defined by the two large masses M_1 and M_2 and beyond M_1. Let r be the distance of the L_3 point from M_2, then

$$\frac{M_1}{(R-r)^2} + \frac{M_2}{(2R-r)^2} = \left(\frac{M_2}{M_1+M_2}R + R - r\right) + \frac{(M_1+M_2)}{R^3}$$

L_4 and L_5 are points that lie on the line defined between the centres of the two masses M_1 and M_2 such that they lie at the third corner of the two equilateral triangles. If a is the radial acceleration, r is the distance from the large body M_1 and sgn (x) is the sign function of x, then

$$a = \frac{-GM_1}{r^2} sgn(r) + \frac{GM_2}{(R-r)^2} sgn(R-r) + \frac{G(M_1+M_2)r - M_2R}{R^3}$$

For the Earth-Sun system, location of L1 is between Sun and Earth and an uninterrupted view of sun is obtained from this point. L2 exists in the same line of Sun and Earth, but beyond Earth (in the opposite direction from Sun). A spacecraft at this point gives a clear view of the deep space as the Sun, Earth and Moon are behind L2. Now, L3 is not yet accessible to us as it is somewhere behind the sun, opposite to the orbit of the Earth. L4 and L5 are stable points and form the Apex of two equilateral triangles with Sun and Earth as the vertices of these triangles. These two points are close to Earth and we can mainly see asteroids and dusts at these points. Asteroids around the L4 and L5 points are known as Trojans.

MAGNETOHYDRODYNAMICS

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The earth we see today was formed from the tiny movements of particles. The whole Universe can be concentrated on a single word "Dynamics". Dynamics is the science that analyses the movement of objects and the forces acting on it. The most relevant branch of it is the fluid dynamics which is the study of properties of fluids in motion. Fluid dynamics has a crucial role in various scientific and engineering field. The notion of, a continuous medium developed by Aristotle, and the physical law of buoyancy initiated by Archimedes was the first step in the study of fluid flow. Sir Isaac Newton's study of fluid statistics and fluid dynamics paved the way for the analysis of fluid flow through mathematical and quantitative physics.

Magnetohydrodynamics is a significant wing of fluid dynamics. It is the physical and mathematical framework that deals with the dynamics of electrically conducting fluids under the influence of a magnetic field. It has raised quite an interest over the years due to its versatile number of applications in various fields; like in geophysics, plasma physics, engineering, biomedical engineering, magnetic drug targeting and many others. Magnetic drug targeting in cancer therapy, which involves establishing more explicit ways to transport cancer-killing drugs to affected areas without harm to surrounding tissues, is one of the significant implementations of magnetohydrodynamics.

The important observations from the historical evolution of magnetohydrodynamics are: Euler (1701-1780) generalized Newton's second law of dynamics to a continuous medium with no internal shear stress and Navier (1785-1836) modified Euler's equation corresponding to uniform viscous fluid. Stokes (1819-1903) presented the concept of internal shear stress, its mathematical frame led to the well-known Navier-Stokes equation. On the other hand, in electromagnetism, Volta (1745-1827) innovated the Voltaic pile or battery. Ampere (1775-1836) found a connection between magnetism and electricity. Later Ohm (1789-1854) established the electrical conduction law and Faraday (1791-1867) discovered that the difference in magnetic flux across a loop generates variation in electric potential. Then Maxwell (1831-189) developed four equations known as Maxwell's equations by extending Faraday's work.

Hartmann in 1937 analysed the steady magnetic flux on moving liquid metal and realized thin boundary layers. Later in 1942, Alfven explained the dynamics of electrically conducting liquids indicated as magnetohydrodynamics or MHD. Hence the exploration of MHD was raised by Hennes Alfven, and Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism are together used to describe magnetohydrodynamics.

Many researchers exploring the MHD flow as it assures highly-valuable potential utilities in various fields

Plasma Physics:

MHD is crucial in understanding and modelling the behaviour of plasmas, which are ionized gases. It helps describe phenomena like magnetic confinement in fusion reactors, where controlled nuclear fusion is sought for energy production.

Astrophysics:

MHD plays a vital role in studying astrophysical phenomena like solar flares, star formation, and accretion disks around black holes. Magnetic fields influence the dynamics of cosmic plasmas, shaping the structures observed in space.

Engineering:

In engineering, MHD finds applications in various fields. For example, it's utilized in designing MHD generators for power generation by converting kinetic energy directly into electrical energy. MHD propulsion systems for spacecraft and magnetic pumps in industrial processes are other engineering applications.

Geophysics:

MHD principles are relevant in Earth's geophysics, contributing to the study of the Earth's magnetic field, dynamo processes, and interactions with the solar wind. Understanding these phenomena is crucial for space weather predictions and related research.

Material Processing:

MHD is employed in metallurgy for processes like electromagnetic stirring in liquid metal, improving the quality of castings. The magnetic field controls the flow and distribution of molten metal, enhancing the overall manufacturing process.

Environmental Monitoring:

MHD sensors can be used in environmental monitoring, measuring parameters like fluid flow and conductivity. This can be applied in water quality assessment or detecting changes in ocean currents.

However, a lot of research work is still required in this field that can make an amazing invention.

A GLIMPSE INTO THE MIND OF A MATHEMATICAL GENIUS

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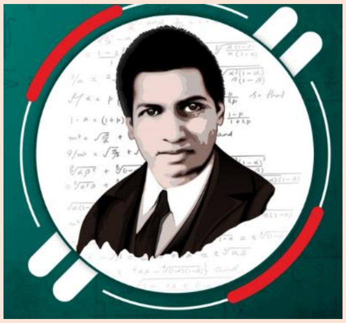

The Child from Tamil Nadu with his Curious Questions at the Age of 11

Srinivasa Ramanujan, a self-taught mathematical prodigy from Tamil Nadu, India, left an indelible mark on the world of mathematics with his extraordinary contributions to number theory, infinite series, and continued fractions. Beyond his formal education limitations, Ramanujan's ability to approach mathematical problems with unparalleled creativity and intuition set him apart. One captivating anecdote that showcases his unique perspective is the fruit division problem, which reflects the genius of a mind that questioned conventional assumptions.

In his youth, Ramanujan's love for mathematics was already evident. The story goes that one day, his mathematics teacher posed a seemingly simple problem to the class: "Suppose you have three fruits in your hand, and you have to divide them among three people. How much will each person get?" While the other students quickly responded with a unanimous "ONE," Ramanujan approached the problem differently. He questioned, "Can there be a fruit if no one is given a fruit?" By doing so, he challenged the conventional thinking and drew a parallel between the problem at hand and the indeterminate form 0/0.

Ramanujan's response to the fruit division problem demonstrated a profound understanding of mathematical concepts, particularly in dealing with indeterminate forms. His ability to think beyond the surface of a problem and question the assumptions involved showcased the depth of his mathematical intuition. The insight behind Ramanujan's question lies in recognizing that if 3/3 is analogous to distributing one fruit to each person, then 0/0 can be considered as a situation where no one receives a fruit. While 3/3 has a clear numerical value, 0/0 is indeterminate, highlighting the unconventional perspective that characterized Ramanujan's mathematical approach.

This anecdote is just a glimpse into the mind of a mathematical genius who would later stun the mathematical community with his prolific work. Ramanujan's approach to problems, like the fruit division scenario, demonstrates the importance of questioning assumptions and thinking outside traditional mathematical boundaries. His unconventional thinking laid the foundation for his groundbreaking contributions and collaborations with eminent mathematicians like G. H. Hardy. Srinivasa Ramanujan's fruit division anecdote encapsulates the essence of his genius – the ability to question, think creatively, and approach mathematical problems from unconventional angles. His story continues to inspire mathematicians worldwide, emphasizing the significance of open-mindedness and creativity in the pursuit of knowledge. The fruit division problem stands as a testament to Ramanujan's exceptional intellect, providing a fascinating insight into the mind of a mathematical prodigy whose legacy endures in the world of mathematics. I hope you all enjoyed this piece from Ramanujan’s life and we have more such stories which you can check out.

UNRAVELING PROBABILITY PUZZLES

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In the realm of probability and decision theory, certain puzzles and paradoxes captivate the minds of enthusiasts and skeptics alike. Two such intriguing challenges are the Monty Hall Problem and the Bertrand's box paradox. These puzzles may appear deceptively simple at first glance, but they unveil the complexities of probability and decision-making.

Monty Hall Problem

Named after the host of the television game show" Let's Make a Deal," the Monty Hall Problem gained fame for its seemingly counterintuitive solution. The scenario is as follows: a contestant is presented with three closed doors. Behind one door is a valuable prize, while the other two conceal goats. The contestant selects one door, after which the host, who knows what is behind each door, opens one of the remaining doors to reveal a goat. Now, the contestant faces a choice: stick with their initial choice or switch to the other unopened door.

Intuition might suggest that since there are only two unopened doors left, the probability of the prize being behind either door is 1/2. However, the counterintuitive truth is that switching doors increases the contestant's chances of winning the prize to 2/3.

The key to understanding the Monty Hall Problem lies in recognizing the host's deliberate revelation of a goat. When the contestant initially picks a door, there is a 1/3 chance that the prize is behind their chosen door and a 2/3 chance that it is behind one of the other doors. If the host opens a door with a goat (which they always do), the probability distribution doesn't change. Thus, switching doors increases the chances of winning.

Bertrand's box paradox

Bertrand's box paradox is a veridical paradox in elementary probability theory. It was first posed by Joseph Bertrand in his 1889 work Calcul des Probabilites. Similar to the Monty Hall Problem, the Bertrand's box paradox challenges our intuition regarding probability.

There are three identical looking boxes in front of you. One of the boxes contains two gold coins, one of the boxes contains one gold and one silver coin and one of the boxes contains two silver coins. You draw a coin from one of the boxes at random and notice that the coin is gold. What is the probability that the other coin in the same box is also gold? This comes down, then, to figuring out the probability that you've picked the gold-gold box instead of the gold-silver box. Many people quickly jump to the conclusion that there are two possibilities, and since the selection was random, it must be 50-50. But this is wrong.

There are three different scenarios in which you can draw a gold coin.

- You draw the gold coin from the box containing 1 gold and 1 silver coin. The probability of the other coin in this scenario being gold is 0.
- You draw gold coin 1 from the box containing 2 gold coins. The probability of the other coin in this scenario being gold is 100%.
- You draw gold coin 2 from the box containing 2 gold coins. The probability of the other coin in this scenario being gold is 100%.

The probability of the second drawn coin being gold is 2/3 as, in two out of three cases, the second drawn coin turns out to be gold.

The Monty Hall Problem and the Three Box Paradox are two provocative examples of how our gut feelings can mislead us when navigating the world of probability. Even though at first glance the situations seem illogical, a deeper look illustrates the value of making educated decisions and the necessity of adjusting probability in light of new knowledge. These riddles, which provide a window into the intriguing realm of probability and decision theory, are still useful resources for scholars, educators, and enthusiasts.

THE MATHEMATICAL FOUNDATIONS OF BITCOIN: A DECENTRALISED REVOLUTION

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Bitcoin is a digital or virtual currency that uses cryptography for security and operates on a decentralized network of computers. It is a form of cryptocurrency, meaning it relies on cryptographic techniques to secure financial transactions, control the creation of new units, and verify the transfer of assets.
Bitcoin, the groundbreaking digital currency that emerged in 2009, has not only disrupted traditional financial systems but also sparked a revolution in the world of mathematics. At its core, Bitcoin relies on a robust cryptographic framework and mathematical principles that underpin its decentralized nature, security, and functionality.
Bitcoin has been a widely discussed and influential cryptocurrency. It's known for introducing the concept of decentralized digital currency and blockchain technology. Bitcoin's value can be highly volatile, influenced by factors such as market demand, adoption, regulatory developments, macroeconomic trends, and overall sentiment within the cryptocurrency community.

Blockchain: The blockchain is a distributed and immutable ledger that contains a record of all Bitcoin transactions. It consists of a chain of blocks, with each block containing a list of transactions. The blocks are linked together using cryptographic hashes, creating a secure and transparent history of transactions.

Limited Supply: Bitcoin has a capped supply of 21 million coins. This scarcity is programmed into the system and is intended to mimic the scarcity of precious metals like gold. The controlled supply is meant to prevent inflammation over time.

Mining and Proof of Work: New bitcoins are created through a process called mining. Mining involves solving complex mathematical puzzles (proof of work) that require significant computational power. Miners compete to solve these puzzles, and the first one to succeed gets the right to add a new block to the blockchain and is rewarded with newly created bitcoins.
Mathematics, especially cryptography and number theory, is essential in ensuring the security and integrity of cryptocurrencies and blockchain technology. Cryptographic techniques, such as public-key cryptography, underpin the secure transfer Bitcoin operates on a decentralized network of computers, commonly referred to as nodes. This means there is no central authority, government, or institution controlling the currency. Transactions are verified by network participants through cryptography and recorded in a public ledger called the blockchain.

Cryptography: The Pillar of Security: At the heart of Bitcoin's security lies cryptography, a branch of mathematics that deals with secure communication. Public-key cryptography, in particular, plays a pivotal role in Bitcoin transactions. Every user is assigned a pair of cryptographic keys - a public key, which is openly shared, and a private key, known only to the owner. This mathematical relationship ensures the integrity and confidentiality of transactions, safeguarding Bitcoin from fraudulent activities.

Elliptic Curve Cryptography (ECC): Bitcoin utilizes Elliptic Curve Cryptography for creating digital signatures, a fundamental aspect of transaction verification. ECC provides a more efficient and secure way to achieve the same level of security as traditional cryptographic systems with much larger key sizes. The compactness and computational efficiency of ECC are crucial for the scalability of the Bitcoin network.

Mining and Proof-of-Work: The process of mining, essential to the creation of new Bitcoin and the validation of transactions, involves solving complex mathematical puzzles. Miners compete to _nd a nonce (a random number) that, when combined with the data in a block, produces a hash that meets certain criteria. This proof-of-work system relies on mathematical algorithms to secure the network, prevent double-spending, and control the issuance of new bitcoins.

Finite Supply and Halving: Bitcoin operates on a fixed supply schedule, governed by mathematical rules encoded in its protocol. The total supply is capped at 21 million bitcoins, creating scarcity and deflationary characteristics. Approximately every four years, a "halving" event occurs, reducing the rate at which new bitcoins are created. This mathematical algorithm controls the issuance of new coins, introducing predictability and scarcity into the Bitcoin ecosystem.

Decentralized Consensus: Bitcoin achieves consensus through a decentralized network of nodes that validate and agree on the state of the blockchain. The consensus algorithm, known as Nakamoto Consensus, relies on mathematical principles to ensure agreement among participants without the need for a central authority. This innovation not only makes Bitcoin resistant to censorship but also eliminates the risk of a single point of failure.

A NOTE ON LATTICE OF TOPOLOGIES

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Lattice theory and topology are two branches of mathematics with origins in the works of two renowned German mathematicians and close friends, Richard Dedekind (1831-1916) and Georg Cantor (1845-1918). Lattices are special partially ordered sets that possess suprema and infima for all finite subsets, demonstrating a relationship between elements in the set. Conversely, a topology on a set X is a collection of subsets of X – referred to as open sets – which include the empty set ϕ and X, and are closed under finite intersections and arbitrary unions. A typical example of a topological space is the real line R with the topology generated by the open intervals of R.

One of the most significant practical applications, and also one of the oldest applications, of modern algebra—especially lattice theory—is the utilization of Boolean algebras in modelling and simplifying switching or relay circuits. Topology finds applications in various branches of mathematics, including differentiable equations, dynamical systems, knot theory, and Riemann surfaces in complex analysis. Moreover, it is employed in string theory in physics for describing the space-time structure of the universe.

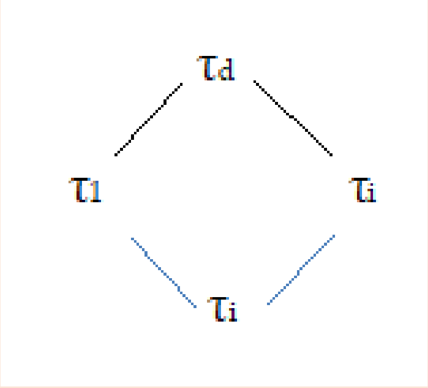
Lattice Theory and Topology are closely connected. For instance, the collection of all open sets of a topological space always forms a lattice. The collection of all topologies on a nonempty set X, denoted $\mathrm{T}\ (X)$, constitutes a complete lattice under set inclusion. This article serves as a brief note on the lattice of topologies on X.

Lattice of Topology

Let X be a nonempty set, and $\mathrm{T}\ (X) = \{\tau \mid \tau \textit{ is a topology on } X\}$. If $\tau_1, \tau_2 \in T\ (X)$, then $\tau_1 \leq \tau_2$ if and only if $\tau_1 \subseteq \tau_2$. Thus, $\mathrm{T}\ (X)$ forms a complete lattice under the partial order relation $\leq$. The least element of the lattice $\mathrm{T}\ (X)$ is the indiscrete topology, $\tau_i = \{X, \emptyset\}$, and the greatest element is the discrete topology $\tau_d = P(X)$. The least upper bound of two topologies τ and τ' is the topology generated by $\{G \cap G' \mid G \in \tau$ and $G' \in \tau'\}$.

Example

Let $X = \{a, b\}$, then $\mathrm{T}\ (X) = \{\ \tau_i = \{X, \emptyset\}, \tau_d = \{X, \emptyset, \{a\}, \{b\}\}, \tau_1 = \{X, \emptyset, \{a\}\}, \tau_2 = \{X, \emptyset, \{b\}\}\}$.



Atomic and Anti-Atomic Properties of $\mathrm{T}(\boldsymbol{X})$

In lattice theory, an atom o is an element that covers the least element, $\tau_i \leq \tau \leq o$ implies $\tau = o$ or $\tau = \tau_i$. An anti-atom is an element that is covered by the greatest element.

The lattice $\mathrm{T}\ (X)$ is an atomic lattice. Every element in $\mathrm{T}(X)$ can be expressed as the join of the atoms $\{\emptyset, X, G\}$, where $\emptyset \subseteq G \subseteq X$. Furthermore, if $|X| = n$, then $\mathrm{T}(X)$ contains $2n - 2$ atoms. In the case where X is infinite, $\mathrm{T}(X)$ contains 2|X| atoms.

If U is an ultrafilter on X and $x \in X$ such that x is not an element in U, then the topology $\tau\ (x, U) = \{G \mid x \notin G\ or\ G \in U\}$ is called an ultra-topology on X. Since these ultra topologies are anti-atoms, and every element in $T\ (X)$ can be expressed as the meet of these anti-atoms, it demonstrates that $\mathrm{T}\ (X)$ is an anti-atomic lattice. If $|X| = n$, then $\mathrm{T}(X)$ contains $n\ (n - 1)$ anti-atoms. In the case where X is infinite, $\mathrm{T}(X)$ contains $2^{2^{|X|}}$anti-atoms.

The study of the lattice of topologies does not end here. It is a broad subject that extends to lattices of T_1, principle, partition, regular, completely regular, and completely accessible topologies.

INTRODUCTION TO BALANCED HYPERCUBES

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Interconnection networks play an essential role in the performance of parallel and distributed systems. Practically, it has been demonstrated that graph theory is a very powerful mathematical tool for designing and analyzing the topological structure of interconnection networks. The architecture of an interconnection network is always represented by a graph, where vertices represent processors and edges represent links between processors. It is impossible to design a network that is optimum from all aspects. One has to design a suitable network depending on its properties and requirements. In the event of practice, large multi-processor systems can also be adopted as tools to address complex management and big data problems. The hypercube network is recognized as one of the most popular interconnection networks, and it has gained great attention and recognition from researchers both in graph theory and computer science. Nevertheless, the hypercube also has some shortcomings. For example, its diameter is large. Therefore, many variants of the hypercube have been put forward to improve the performance of the hypercube in some aspects. Among these variants, balanced hypercube has desirable properties of strong connectivity, regularity, and symmetry. The proposed structure is a special type of load-balanced graph designed to tolerate processor failure. This was first introduced by Huang and Wu. For balanced hypercubes, each processor has a backup (matching) processor that shares the same set of neighbouring nodes. Therefore, tasks that run on a faulty processor can be reactivated in the backup processor to provide efficient system reconfiguration. It is also shown that odd dimensional balanced hypercubes have smaller diameters than that of standard hypercubes.

An n −dimensional balanced hypercube BH_n consists of 2^{2n} vertices $(a_0, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n)$, where $a_i \in \{0, 1, 2, 3\}$ $(0 \leq i \leq n-1)$. An arbitrary vertex v $=(a_0, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n)$, in BH_n has the following $2n$ neighbors:

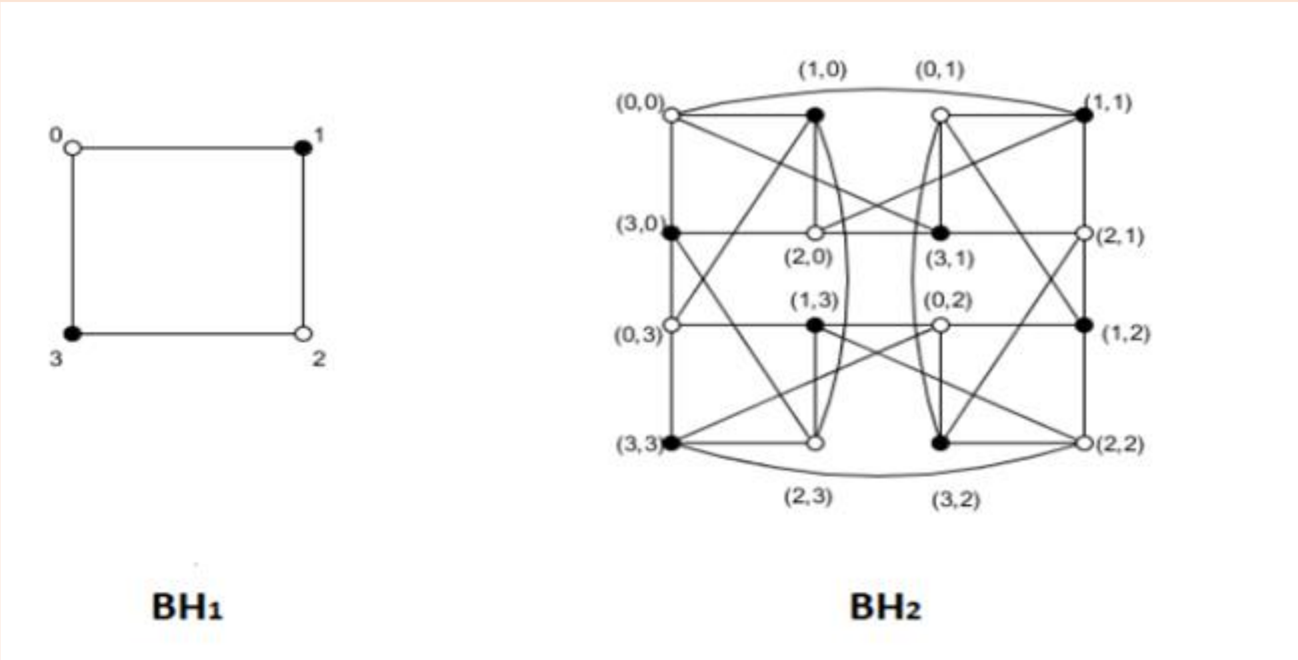
- $((a_0+1) mod\ 4, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1}))$,
  $((a_0-1) mod\ 4, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1}))$, and
- $((a_0+1) mod\ 4, a_1, \ldots, a_{i-1},\ ((a_i+1)\ mod\ 4), a_{i+1} \ldots, a_{n-1})$,
  $((a_0-1) mod\ 4, a_1, \ldots, a_{i-1},\ ((a_i+1)\ mod\ 4), a_{i+1} \ldots, a_{n-1})$,

In BH_n, the first coordinate a0 of vertex $(a_0, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n)$, is named the inner index, and the other coordinates ai $(0 \leq i \leq n-1)$ are i −dimensionindices. Clearly, every vertex in BH_n has two inner adjacent vertices and $2n-2$ other adjacent vertices. Wu and Huang gave an equivalent definition of BH_n, which can be hierarchically constructed as follows:

1. BH_1 is a 4-cycle, the vertices of which are labeled as 0, 1, 2, 3, respectively.

2. BH_{k+1} is constructed from 4 $BH_K{}'s$. These four $BH_K{}'s$ are labelled $BH_K(0)$, $BH_K(1)$, $BH_K(2)$, $BH_K(3)$ where each vertex in $BH_K(i)$ $(0 \le i \le 3)$ has i attached as the new $k-th$ index. Every vertex $v = (a_0, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_n)$, in $BH_k(i)(0 \le i \le 3)$ has two extra neighbors:

- $((a_0+1) mod\ 4, a_1, \ldots, a_{k-1}, (i+1)\ mod\ 4))$,
  and $((a_0-1) mod\ 4, a_1, \ldots, a_{k-1}, (i+1) mod\ 4))$, which are in $BH_k(i+1)$ if a_0 is even.
- $((a_0+1) mod\ 4, a_1, \ldots, a_{k-1}, (i-1)\ mod\ 4))$,
  and $((a_0-1) mod\ 4, a_1, \ldots, a_{k-1}, (i-1) mod\ 4))$, which are in $BH_k(i-1)$ if a_0 is odd.



Similarly, we can construct for higher dimensions. Like hypercubes, the balanced hypercubes are bipartite, vertex-transitive, and edge-transitive. It has many interesting properties like edge-pancyclicity, bi-panconnectivity, super (edge) connectivity, matching extendability, extra connectivity, Hamiltonian laceability, hyper-Hamiltonian laceability, and so on. BH_n has the matching preclusion number $2n$, and the conditional matching preclusion number is $4n - 2$ whenever $n \ge 2$. Other properties of balanced hypercubes are being examined.

REVIEW OF THE PAPER "A FORMAL MODEL OF SYSTEM C COMPONENTS USING FRACTAL HYPERGRAPHS"

Deepthi Chandran, Research Scholar, Department of Mathematics, RSET

Nicolas Vallee and Bruno Monsuez introduced a new mathematical structure known as Fractal Hypergraphs. This structure was used to abstract the object-oriented nature of System C language. The object-oriented nature of system C is similar to the hierarchical and compositional nature of fractal hypergraphs.

In the introduction section presented the key aspects of System C that should be captured and fully supported by the formal representation of System C components.

1. The formal representation copes with different abstraction levels; it also must support the refinement.
2. The formal representation must support program code and hardware description; a semantics for program execution and a semantics for system simulation should be provided.
3. The formal representation must support assembling modular and parametrized components.

Due to the hierarchical nature, instead of graphical representation, an extension of graphs called fractal hypergraphs is necessary. Hence in section 2 fractal hypergraphs is defined. Directed hypergraph is defined here as there is a transition between an initial and a final state. They defined fractal hypergraphs as follows:

A fractal hypergraph is defined as:

- a basic fractal hypergraph$HB = \left(V, H, \varepsilon_{in}, \varepsilon_{out,}\varepsilon\right)$
- a set of entry vertices in_V, a set of exit vertices V_{out}
- a set of entry edges ∂_{in}- a binary relation $\partial_{in} \in \ \delta(V_{in} \times V)$ that connects the entry vertices V_{in} to the vertices of the basic fractal hypergraph V.
- a set of entry edges ∂_{out}- a binary relation $\partial_{out} \in \ \delta(V_{out} \times V)$ connecting the exit vertices V_{out} to vertices of the basic fractal hypergraph V.

They mentioned the properties of new structure as follows:

Multiple hierarchy

The model supports substituting a component with another component; a fractal hypergraph models a component which is replaced by another fractal hypergraph representing another component. This preserves the abstract properties.

Concurrency

A hyperedge that connects a set of initial states to a set of final state may cover two or more concurrent transition, each of those transitions are represented by sub hyperedge which is located in the fractal subhypergraph that describes the hyperedge.

Aggregation and Parametrization

Components are represented by fractal hypergraphs. Components may statistically aggregated subcomponents, represented by a subfractalhypergraph. Each time a parameter is required, an empty subgraph is inserted in the fractal hypergraph. When aggregating the parametrized fractal hypergraph, the empty subhypergraphs get replaced by the hypergraphs that denote the selected implementation.

In the same section a semantics is described based on fractal hypergraphs. The structure and transformation of fractal hypergraphs define an underlying semantics. The structure of fractal hypergraphs represents all the history of execution. Fractal hypergraphs represent values, function closures as well as objects. An evaluation of such a fractal hypergraph returns the value associated with it. The semantic rules can be concurrent execution or conditional branching.

In section 3 System C component is represented by fractal hypergraphs. This includes classes, objects as well as template and template intantiation.

Finally in last section (section 4) authors showed in detail the connection between System C components or the association of methods defined in system C components to an event translate into the fractal hypergraph-based representation. Thus, the representation using fractal hypergraph explored to many applications.

By referring this paper, Adam Obtulowicz published a paper named "In search of a Structure of Fractals by Using Membranes as Hyperedges."

UNLOCKING DATA PATTERNS: MAPPER ALGORITHM AND TOPOLOGICAL DATA ANALYSIS

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The world we live in is characterised by the extraordinary rate at which different types of data are being produced. We have many kinds of data in different fields, from basic spreadsheets to complicated data from social networks, historical archives, genetic sequencing, medical records, and brain networks, among other sources. Not only is data growing in volume these days, but it's also becoming more noisy, unstructured, dynamic, and less static. Only until we can assess and extract the relevant information from data does it become useful. Thus, a method for visualising huge, complicated data sets with high dimensions are required from which information must be extracted. Evidently, we are falling behind in our ability to evaluate this data in terms of its amount and nature.

The T.D.A seed

Russia's Konigsberg city was situated on both banks of the Pregel river. It consisted of the two sizable islands, Kneiphof and Lomse, which were connected to the two parts of the city that were on the mainland by seven bridges. The challenge was to walk over every bridge in Konigsberg, although just once, and still arrive at one's starting place. Leonard Euler, an 18th-century mathematician, simplified the problem to one involving nodes and lines: the four land sections were the nodes, connected by bridges, which were represented by lines.He discovered that an even number of bridges would be required in each land region so that each could be crossed just once.

Topologists discovered several things from the puzzle, one of which being that the answer did not depend on the precise locations, lengths, or depths of the bridges or the river. The quantity and configuration of the bridges were the only factors that mattered. That is where the idea for Topological Data Analysis (TDA) originated. This method was developed by Stanford University mathematician Gunnar Carlsson, who used distance as an input that translates into a topological shape or network. The method represents complex big data sets as networks of nodes and edges, creating an intuitive map of data based solely on similarity of data points.

In the final map, the data points will be arranged more or less in line with one another depending on how similar or distinct they are from one another. The core of TDA is this. The idea of TDA led to a number of fascinating discoveries, one of them being persistence diagrams. A connected graph comprising of extensive datasets can be created by Gurjeet Singh's MAPPER algorithm, which he developed in 2007. The functioning of this algorithm will be examined in this paper. As a topological description of the point cloud data under analysis, MAPPER produces a graph with nodes and edges. Beyond everything else, it generates a connected graph that visually represents the whole set of data Mapper in detail.

A graph with nodes and edges that provide a topological summary of the point cloud data under analysis is the result of MAPPER. The following is how Mapper's concept is put forth: Let's say we have point cloud data that depicts a form. To minimise complexity through dimensionality reduction, we project on a coordinate and colour the data points based on filter values. The point cloud data is now divided into overlapping bins because it is covered in overlapping intervals. The points in each bin are then collapsed into clusters using a clustering method. Once clustering is done, we can then create a network where each cluster of a bin is represented by a vertex and we can draw an edge when there is non-empty intersection between clusters.

The above said is an overall idea of what happens in a mapper algorithm. To get a clear picture one has to know about various filters used in the algorithm.

i) Filter function: A function that maps a data frame X to the real numbers, that is $f : X \to R$. This is used to cover the data set with overlapping sets.

ii) Clustering technique: A technique which allows the grouping of data entries into meaningful subsets of the data.

iii) Number of Intervals: the total number of subsets which create an open cover of $f(X)$.

iv) Percent Overlap: The percent overlap of each open cover in $f(X)$.

The following examples will give an overall picture.

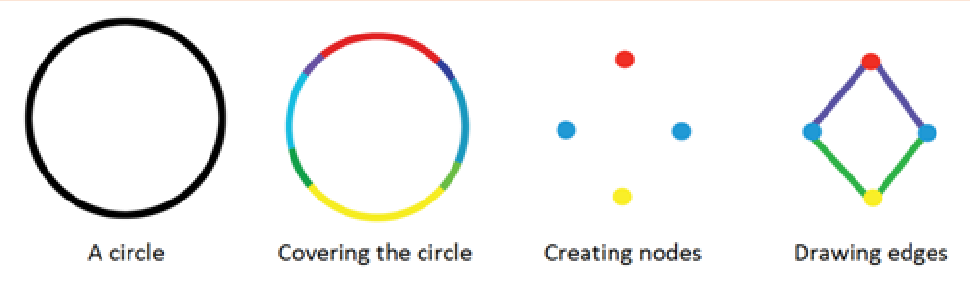
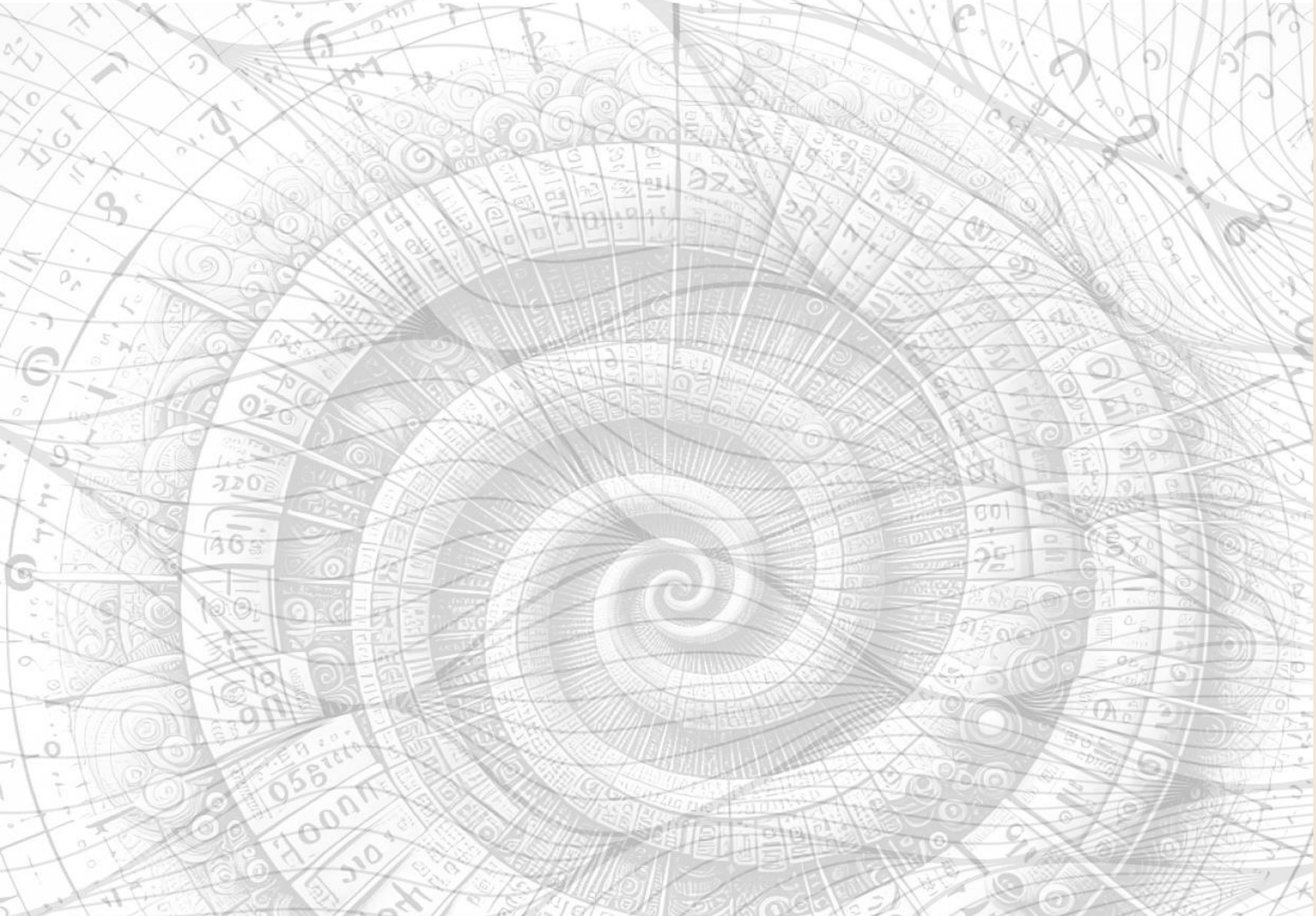


Figure 1: The mapper approach is applied to a circle; first the circle is covered by the values of the chosen filter function, then the range of those filter values are formed into overlapping intervals and finally nodes are created and an edge is drawn between two nodes if they have at least one element in common.

The Mapper Algorithm in Topological Data Analysis adapts to users' needs, accepting varied inputs for extracting specific information. Choosing the right filter function is pivotal yet challenging, with no defined method to determine the most suitable one. Parameters like clustering techniques, intervals, and overlap is user-dependent. Carlsson and collaborators applied Mapper to a breast cancer genomics study, transforming spreadsheet data into a network. The resulting "map" revealed a distinct Y-shape, clustering patients based on survival. This visual aided geneticists in pinpointing influential genes.

TDA aims to shape data for insightful analysis. Mapper summarizes large datasets into a graph, enabling visualization of variables often missed by traditional statistics. However, Mapper's dependence on multiple inputs can lead to differing conclusions from the same dataset, highlighting its weakness. Understanding how inputs affect the overall summary is key to mitigating this. Future research could refine filter function choices and tweak the algorithm for diverse scenarios.



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